Updating Circuit Theory: Relating Photons to Charges

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Summary

A definitive relationship is established between photons and charges. Charges flow axially along the send conductor and radially towards the surface. At the surface, they are stopped by atomic forces. Photons are emitted. The rate of change of the number of departing photons is proportional to the number of charges arriving at the surface. The energy lost to the conductor is measured by the voltage developed along the inductance. The energy stored in the conductor is measured by the number of charges held at the surface. This assessment was obtained by successive refinements of the transmission line model of the differential-mode current waveform in a twin-conductor cable which is open-circuit at the far end.

Introduction

A test is described where a step voltage is applied to the near end of a twin conductor cable which is open-circuit at the far end and a photographic record is taken of the differential-mode current. Then a circuit model is created which replicates the recorded waveform. Analysis of the model provides an insight into the mechanisms involved.

Details are provided of the setup, the test method, and the test result; a waveform with multiple discontinuities. Then a circuit model is created to replicate that waveform. Details are provided of the reasoning used to create that model. A Mathcad worksheet is developed to simulate the recorded waveform. Details are provided of the values assigned to the constants and all the variables are defined. The purpose of each function (subroutine) is described and a block diagram provided of the computation process.

The end result is a waveform which correlates quite closely with that created by the test equipment. Assessment of the results provides a definitive relationship between the movement of charges along the conductor and the propagation of photons to and fro between the conductors.

Since the terminals at the far end are open-circuit, the eventual state is a constant voltage between the conductors due to charges trapped on the surfaces. But this is sustained by photons which continue to propagate to and fro between the conductors.

Setup

In the setup illustrated by Figure 1, the signal generator was set to provide a square wave voltage to a potential divider which allows the resistance of the source applied at the near end terminals of the cable to be approximately 9 ohm. This means that every transient step will cause a step current to propagate along the cable, be reflected at the open-circuit terminals at the far end of the line and be reflected again by the low resistance at the near end. Textbook theory indicates that the waveform of the current in the line would be a square wave which gradually reduces in amplitude.

The differential-mode current was measured by an oscilloscope connected to a current transformer. Details of this transformer are provided by the article 'Updating Circuit Theory: The Current Monitor Transformer'.

A common-mode choke is used to minimise the common-mode current flowing between the cable and the signal generator.

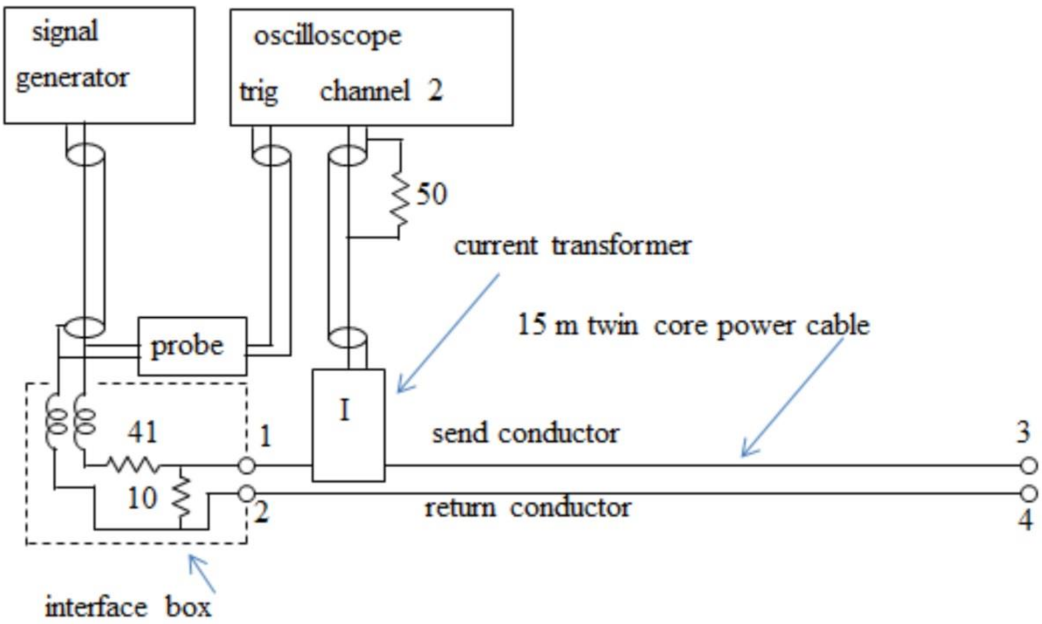


Figure 1 Measuring the response of a twin core cable

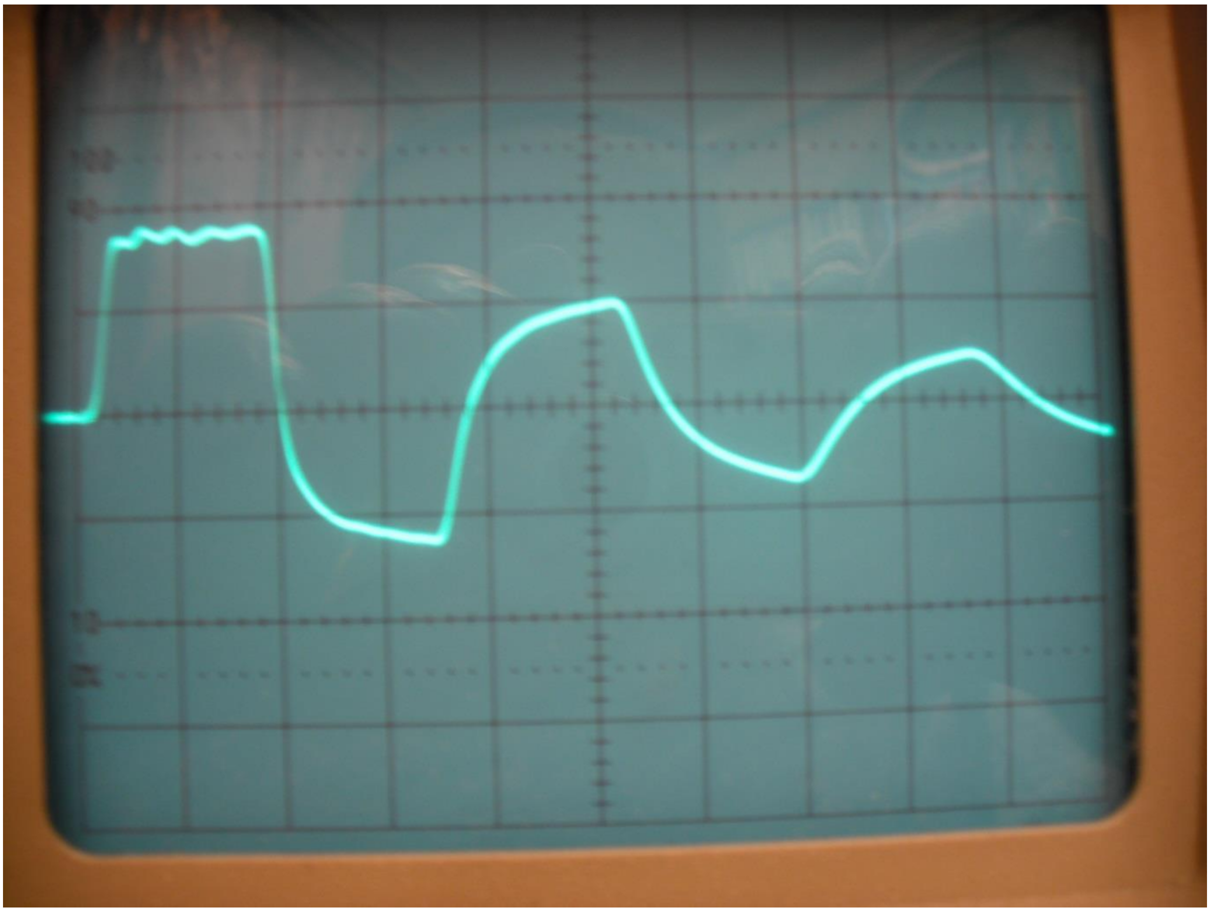
Test Method

By selecting the operating frequency of the signal generator to be much lower than the frequency of the line transients, the response of the line to a single voltage step can be measured. Enough time between the rising and falling edges of the signal generator output is provided to allow the transients in the cable to decay to an imperceptible level.

Once the frequency of the square wave output of the signal generator had been selected, a voltage probe was used to measure the peak-to-peak amplitude of the voltage applied to the input terminals of the cable.

Then the waveform of the differential waveform was displayed on the oscilloscope. The use of an input trigger was used to lock the scan time to the source waveform.

Photographs were taken of the current waveform, and one of these is shown by Figure 2.



Horizontal scale: 100ns/div Vertical scale: 10mV/div

Figure 2 Waveform of the differential-mode current

Basic Model

The model illustrated by figure 3 allows the basic parameters to be related to currents and voltages at the line terminations

Vgen is the source voltage
Rg is the source resistance at the near end
RL is the load resistance at the far end

T is the time taken for a signal to propagate from the near end to the far end.
Ro is the characteristic resistance of the differential-mode loop

Ina is the loop current flowing at the near end.
Vna is the voltage induced at the near end by the transients in the line.
Ini and *Inr* are the incident and reflected currents at the near end.

Ifa is the loop current flowing at the far end.

Vfa is the voltage induced at the far end by the transients in the line.
Ifi and *Ifr* are the incident and reflected currents at the far end.

The relationships between these parameters are derived in the article 'Updating Circuit Theory: Charges and Photons'. A simulation of this model would result in a square wave of gradually decreasing amplitude.

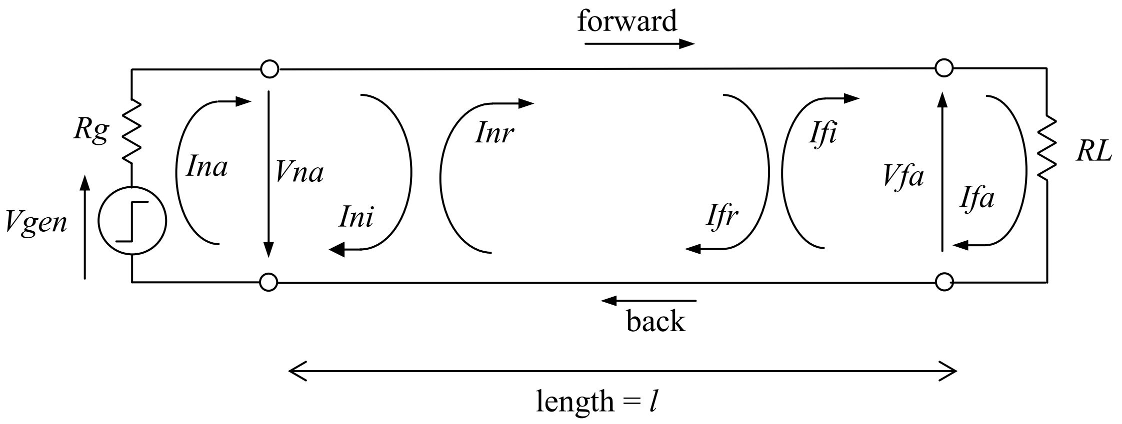


Figure 3 Basic model of transmission line: defining the parameters

The waveform of Figure 2 identifies features of the signal that are not accounted for in these basic relationships. But the nature of these features provides significant clues as to what is happening as the front edge of the step as it propagates forward and backward along the cable.

During the first transit forward and the first transit back the current delivered to the line is constant. The ripples are probably due to discontinuities in the layout of the cable. The finite time of the leading edge is due to the reaction time of the current transformer and the frequency response of the oscilloscope.

Since the first trailing edge of the waveform resembles that of an R-C network which goes open-circuit, such a circuit was incorporated into the model. The nett result was a square wave which gradually morphs into a sinusoidal waveform. But it did not replicate the discontinuities which occur during each half-cycle of Figure 3.

Antenna-Mode Current

In the setup of Figure 1 it is clear that the current monitored by the current transformer is the differential-mode current. Any current which flows along both conductors in parallel would not be detected. That is, antenna-mode current which flows forward along the cable and out into the environment would have no effect on the waveform displayed by Figure 2.

The differential mode current is that which flows along the send conductor and back along the return conductor. If some of the field emanating from the send conductor during forward propagation creates a current which flows forward along the return conductor, then this current would not be detected either.

It was reasoned that the exponential decay could also be due to inductive effects. The current at the far end rises exponentially with time. The missing portion of the original step function could still be due to antenna-mode emission. This reasoning led to the replacement of the R-C circuit with the L-R circuit illustrated by Figure 4a. This model provides the loop equation:

$$Ro \cdot Inr = Ro \cdot Int + Lrad \cdot \frac{dInt}{dt} \tag{1}$$

Where *Int* is the amplitude of the current arriving at the far end of the send conductor and *Lrad* is the radiation inductance. If *Int* is the current transmitted to the far end, then the current lost to the differential-mode loop during the transit from near end to far end must be:

$$Ins = Inr - Int \tag{2}$$

The current lost to the differential-mode loop is stored in the antenna-mode loop and this can also be calculated using the model of Figure 4b. The loop equation here is:

$$Ro \cdot Inr = Ro \cdot Ins + \frac{Qns}{Crad} \tag{3}$$

Where *Crad* is the radiation capacitance and

$$Qns = Ins \cdot dt \tag{4}$$

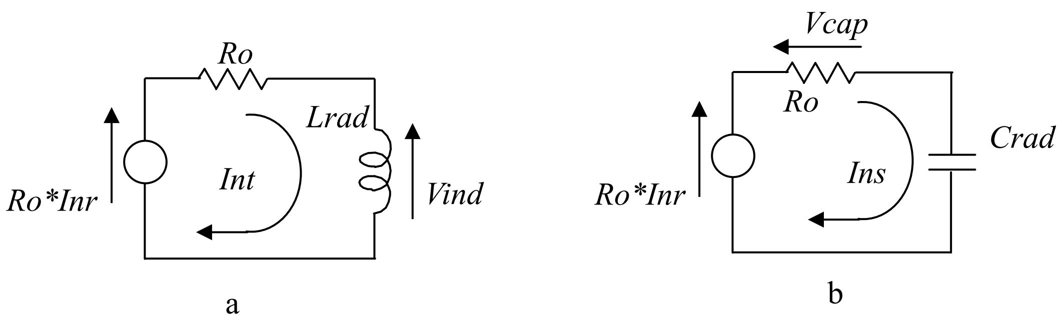


Figure 4 Modelling antenna-mode current
a calculating transmitted current
b calculating stored current

It is possible to simulate the exponential fall in the antenna-mode current at the far end using either Figure 4a or Figure 4b. The computer program used to simulate this effect is based on the model of Figure 4a; that is, on equations (1) and (2). The use of an inductor to simulate the rise time at the far end allows it to be reasoned that some of the missing current could be stored in the return conductor.

Antenna-Mode Reflection

Photons departing from the surface of the send conductor and arriving at the surface of the return conductor will induce charges which can flow in both directions. Some will be reflected back to the send conductor and will enhance the forward flow in the send conductor.

Charges which flow forward along the return conductor are propagating in the same direction as those in the send conductor. This creates antenna-mode current, and photons created by this current propagate out into the environment.

When the leading edge of the step arrives at the far end of the cable, the return conductor is carrying approximately half of the antenna-mode current. Conversely, the current arriving at the end of the send conductor is zero. Current arriving at an open circuit terminal immediately reverses in direction. So now it is the return conductor which is delivering antenna-mode current to the send conductor.

The differential-mode current arriving back at the near end consists of two components; that which has been reflected back from the far end, and the proportion *k* of the antenna-mode current delivered back from the return conductor.

This mechanism can be simulated by the worksheet functions defined by the block diagram of Figure 5.

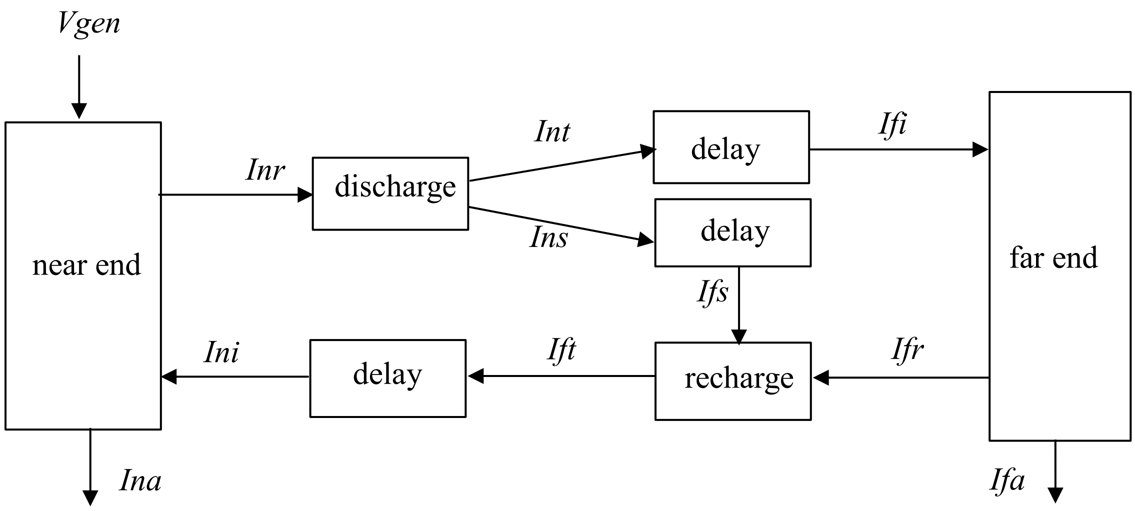


Figure 5. Signal processing functions for worksheet

Worksheet Constants

A copy of the first page of the worksheet is provided by Figure 6. The values of the input parameters need some explanation.

The amplitude of the voltage step between terminals 1 and 2 of Figure 1, as monitored by the voltage probe, was

$$Vin = 0.84 \ \text{Volt} \tag{5}$$

Examination of the amplitude of the first step in the waveform of Figure 2 gave a value for *Vch2* of 17 mV. The relationship between this voltage and the amplitude of the current monitored by the current transformer is *RT*, the transfer resistance. The value of this parameter has been measured to be 2.27 ohm. Details of the calibration of this transformer can be found in the article 'Updating Circuit Theory: The Current Monitor Transformer'. Hence the value of the differential-mode current flowing in the cable was

$$Idiff = \frac{Vch2}{RT} = 7.49 \ \text{mA} \tag{6}$$

So, the characteristic resistance *Ro* of the cable was

$$Ro = \frac{Vin}{Idiff} = 112 \ \text{ohm} \tag{7}$$

The value of the source resistance *Rg* was determined by examination of the circuitry of the interface box of Figure 1.

$$Rg = \frac{10 \cdot (41+50)}{10+41+50} = 9 \ \text{ohm} \tag{8}$$

The voltage *Vg* necessary to deliver *Vin* to the terminals at the near end must be

$$Vg = Vin \cdot \frac{Rg + Ro}{Ro} = 0.91 \ \text{Volts} \tag{9}$$

The terminals at the far end of the cable are open circuit. Setting the value of the load resistance *RL* at 100 M-ohm allows this interface to be simulated.

Figure 2 can be used to define the time *T* taken for the step to propagate along the cable. The time between the leading and trailing edges of the first pulse is that taken for two traverses; forward and back. Inspection of Figure 2 gives

$$T = \frac{185-20}{2} = 82.5 \ \ \text{nano-seconds}$$

In this simulation, the number of segments *n* into which the line is divided is 100. So the time *dt* for the edge to propagate along one segment is

$$dt = \frac{T}{n} = 825 \ \text{pico-seconds} \tag{10}$$

An approximate value for the time constant *Trad* of the exponential decay of the trailing edge can be deduced from Figure 2. Assigning an approximate value to this parameter allows the inductance *Lrad* of the antenna-mode loop to be calculated.

$$Lrad = Ro \cdot Trad \tag{11}$$

Successive iterations of the program allow the value of the time constant to be refined. (*Trad* is the only select-on-test parameter)

T1 is the time delay between the start of the scan and the first leading edge. *T2* is the duration of the scan.

Worksheet Functions

In previous simulations, a shift register was used to simulate the propagation of charges from one segment of the line to the next. So the entire contents of the register changed at each step. In this simulation, the only change is to the value of a single segment. Pointers are used to identify that segment. If *i* is the number of the iteration, the number *n* of segments in the register is ten, then the relationship between iteration and pointer is defined by Table 1.

The pointer *p1* identifies the segment of the ***Forward*** register which is to be updated and *p2* identifies the associated segment of the ***Back*** register.

The function *point* (*i*) calculates the values of *p1* and *p2* for any value of *i*. (The Mathcad function *mod* (*i* , *n*) returns the remainder on dividing *n* by *i*) .

i	1	2	3	4	5	6	7	8	9	10
i	11	12	13	14	15	16	17	18	19	20
i	21	22	23	24	25	26	27	27	29	30
i	31	32	33	. . .	. . .	. . .				
p1	1	2	3	4	5	6	7	8	9	10
p2	10	9	8	7	6	5	4	3	2	1

Table 1 Relating the segments of the registers to the number of time steps

The function *delay* (***D*** , *In* , *p*) records the value of the element *p* of the register ***D*** as *Iout.* Then it updates that element with the value of *Iin.* The output of the function is the value *Iout.* This value is that which was entered during the previous scan. Since a complete scan represents a time delay of *T* seconds, the input is the value of *Iin* at one end of the line and the output *Iout* is the value at the other end.

The functions *near* (*Ini* , *Vg*) and *far* (*Ifi*) have been explained in previous articles.

The function *charge* (*Inr* , *Int*) uses equations (1) and (2) to calculate the amplitudes of the current *Int* transmitted to the far end and the current *Ins* stored in the antenna-mode loop.

During the first half cycle, the current *Ins* simulates the total current radiated away from the send conductor. Half radiates away from the outward-facing surface in the form of photons departing at the velocity of light. Photons departing from the surface facing the return conductor create antenna-mode current in that conductor.

The antenna-mode current arriving at the far end of the return conductor must be half the value of *Ifs*. The antenna-mode current arriving at the far end of the send conductor must be zero, since only half can be collected by the return conductor. Any current arriving at an open-circuit terminal is immediately reflected back. So this reflected current reappears as differential-mode current.

Hence, the total amplitude of the differential-mode current reflected back from the far end is:

$$Ift = Ifr - k \cdot Ifs \tag{12}$$

where $k = 0.5$.

Main Program

The main program is shown on Figure 7.

The control variable *i* is set to step between unity and *N2*; the number of steps needed to simulate a single sweep of the oscilloscope.

The number of elements in each register is defined as *n*, the number of segments into which the cable is divided. The register ***Antenna*** holds the values of the parameter *Ins*, the current 'stored' in the antenna-mode loop.

The function *point* (*p1*, *p2*) defines the value of each pointer at each instant. The amplitude of the voltage source *Vgen* is held at zero until the time *T1* corresponds to that of the leading edge of the recorded waveform. Then the sequence of operations follows that of the block diagram of Figure 5.

The output vector ***Out*** can collect the value of any parameter in Figure 5 at every step of the program. In the worksheet of Figure 7, this is the value corresponding to the amplitude of the waveform displayed on the scope: the current at the near end *Ina*, as monitored by the current transformer, multiplied by the transfer resistance *RT*

Assessment

The waveform of Figure 7 now bears a close resemblance to that of the configuration-under-review. Since the frequency response of the oscilloscope is about 20 MHz, the spot on the screen cannot move as fast as the rate of change of the current being monitored. This is evident in the finite rise time on the leading edge and the finite fall time of the first trailing edge of Figure 2.

The significant feature of the simulated waveform is that it replicates the kinks in the actual response. These discontinuities are visible at every peak of Figure 7.

$Vg := 0.91$ $\quad Ro := 112$ $\quad Rg := 9$ $\quad RL := 100 \cdot 10^{6}$ $\quad RT := 2.27$

$T := 82.5 \cdot 10^{-9}$ $\quad n := 100$ $\quad dt := \frac{T}{n}$

$Trad := 32 \cdot 10^{-9}$

$Lrad := Trad \cdot Ro = 3.584 \times 10^{-6}$ $\quad k := 0.5$

$T1 := 20 \cdot 10^{-9}$ $\quad N1 := floor\left(\frac{T1}{dt}\right) = 24$

$T2 := 1000 \cdot 10^{-9}$ $\quad N2 := floor\left(\frac{T2}{dt}\right) = 1.212 \times 10^{3}$

$$point(i) := \left| \begin{array}{l} p1 \leftarrow mod(i, n) \\ p1 \leftarrow n \ \ if \ \ p1 = 0 \\ p2 \leftarrow n - p1 + 1 \\ (p1 \quad p2) \end{array} \right.$$

$$delay(\mathbf{D}, Iin, p) := \left| \begin{array}{l} Iout \leftarrow \mathbf{D}_p \\ \mathbf{D}_p \leftarrow Iin \\ (\mathbf{D} \quad Iout) \end{array} \right.$$

$$near(Ini, Vg) := \left| \begin{array}{l} Ina \leftarrow \dfrac{2 \cdot Ro \cdot Ini + Vg}{Ro + Rg} \\ Inr \leftarrow Ina - Ini \\ (Inr \quad Ina) \end{array} \right.$$

$$charge(Inr, Int) := \left| \begin{array}{l} dInt \leftarrow \dfrac{dt}{Lrad} \cdot Ro \cdot (Inr - Int) \\ Int \leftarrow Int + dInt \\ Ins \leftarrow Inr - Int \\ (Int \quad Ins) \end{array} \right.$$

$$far(Ifi) := \left| \begin{array}{l} Ifa \leftarrow \dfrac{2 \cdot Ro \cdot Ifi}{Ro + RL} \\ Ifr \leftarrow Ifa - Ifi \\ (Ifr \quad Ifa) \end{array} \right.$$

$$recharge(Ifs, Ifr) := \left| \begin{array}{l} Ift \leftarrow Ifr - k \cdot Ifs \\ Ift \end{array} \right.$$

Figure 6 First page of Mathcad worksheet. Input variables and processor functions

$i := 1 .. N2 \qquad\qquad t_i := i \cdot dt$

$$
\mathbf{Out} := \left|
\begin{array}{l}
\mathbf{Forward}_n \leftarrow 0 \\
\mathbf{Back}_n \leftarrow 0 \\
\mathbf{Antenna}_n \leftarrow 0 \\
\text{for } i \in 1 .. N2 \\
\quad \left|
\begin{array}{l}
(p1 \;\; p2) \leftarrow point(i) \\
Vgen \leftarrow Vg \;\; \text{if } i > N1 \\
(Inr \;\; Ina) \leftarrow near(Ini, Vgen) \\
(Int \;\; Ins) \leftarrow charge(Inr, Int) \\
(\mathbf{Forward} \;\; Ifi) \leftarrow delay(\mathbf{Forward}, Int, p1) \\
(\mathbf{Antenna} \;\; Ifs) \leftarrow delay(\mathbf{Antenna}, Ins, p1) \\
(Ifr \;\; Ifa) \leftarrow far(Ifi) \\
Ift \leftarrow recharge(Ifs, Ifr) \\
(\mathbf{Back} \;\; Ini) \leftarrow delay(\mathbf{Back}, Ift, p2) \\
\mathbf{Out}_i \leftarrow Ina \cdot RT
\end{array}
\right. \\
\mathbf{Out}
\end{array}
\right.
$$

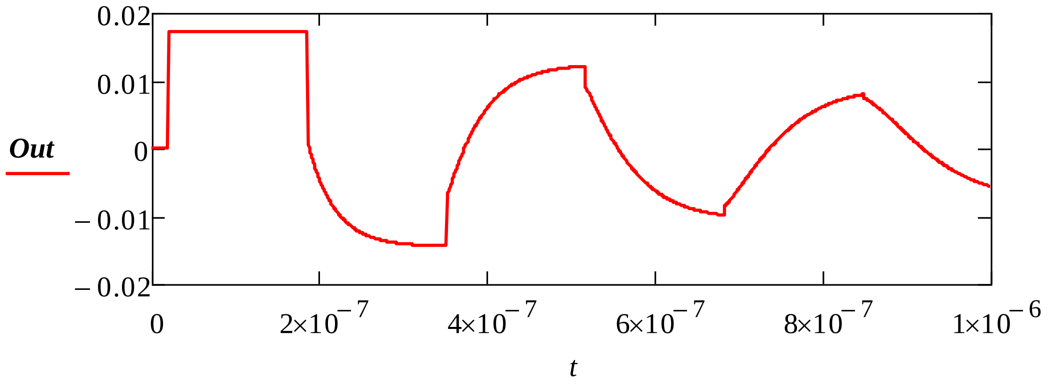


Figure 7 Second page of worksheet. Definition of time steps, and main program,

Alternative Method

It is possible to create exactly the same simulation by replacing the function *charge*(*Inr* , *Int*) on the first page of the worksheet with that shown on Figure 8, where

$$Crad = \frac{Trad}{Ro} \tag{13}$$

This means that the circuit models of Figures 4a and 4b are equally valid

$$charge(Inr, Qns) := \left| \begin{array}{l} Ins \leftarrow Inr - \dfrac{Qns}{Ro \cdot Crad} \\ Int \leftarrow Inr - Ins \\ Qns \leftarrow Qns + Ins \cdot dt \\ (Int \quad Ins \quad Qns) \end{array} \right.$$

Figure 8 Alternative method of calculating components of antenna-mode current

Charges and Photons

The classical analysis of the transient responses of Figures 4a and 4b are

$$Int = \frac{Vnr}{Ro} \cdot \left(1 - e^{-\theta}\right) \tag{14}$$

and

$$Ins = \frac{Vnr}{Ro} \cdot e^{-\theta} \tag{15}$$

where the time constant *Trad* is

$$Trad = \frac{Lrad}{Ro} = Crad \cdot Ro \tag{16}$$

and the angle θ is the ratio

$$\theta = \frac{t}{Trad} \tag{17}$$

Differentiating equation (14)

$$\frac{dInt}{dt} = \frac{Vnr}{Ro} \cdot e^{-\theta} = Ins = \frac{dQns}{dt} \tag{18}$$

This relationship indicates that *Inr* and *Ins* are flowing in opposite direction and that *Ins* is equal to the rate of change of the charge *Qns*. It correlates with the assumption that the rate at which photons are emitted from the surface is proportional to the sudden deceleration of charges as they arrive at the surface of the conductor.

It can also be inferred that the rate at which photons arrive at the surface of a conductor is equal to the amplitude of the current delivered to that surface.

Magnetic Field

The work done in ejecting photons from the surfaces of the conductors is the voltage *Vind* depicted in Figure 4a. The work done in driving charges to the surfaces of the conductors is *Vcap* as depicted in Figure 4b.

This means that the energy *Vcap* needed to hold charges at the surfaces of the conductors is the same as the energy *Vind* involved in transmitting photons into the environment; that is, the same as the energy of the magnetic field.

Electric Field

The differential-mode voltage at the output terminals *Vout* can be calculated using the relationship

$$Vout = RL \cdot Ifa \tag{19}$$

The waveform settles down into 'steady-state' conditions after about 5 micro-seconds, as shown by Figure 9. Even so, the photons continue to propagate to and fro between the conductors. This is sustained by charges on the surfaces of both conductors. That is, the energy is now trapped at each surface by atomic forces. The atoms must be vibrating at an extremely high frequency. The actual state of the system is anything but stationary. But the number of photons in motion is orders of magnitude less than that associated with the magnetic field.

It is obvious that the voltage at the far end is equal to the open-circuit voltage *Vg*. So there is no voltage drop in the source resistor *Rg*. This can only happen if the differential-mode current is infinitesimal. If the amplitude of the current is minute, then the number of photons it ejects must also be tiny.

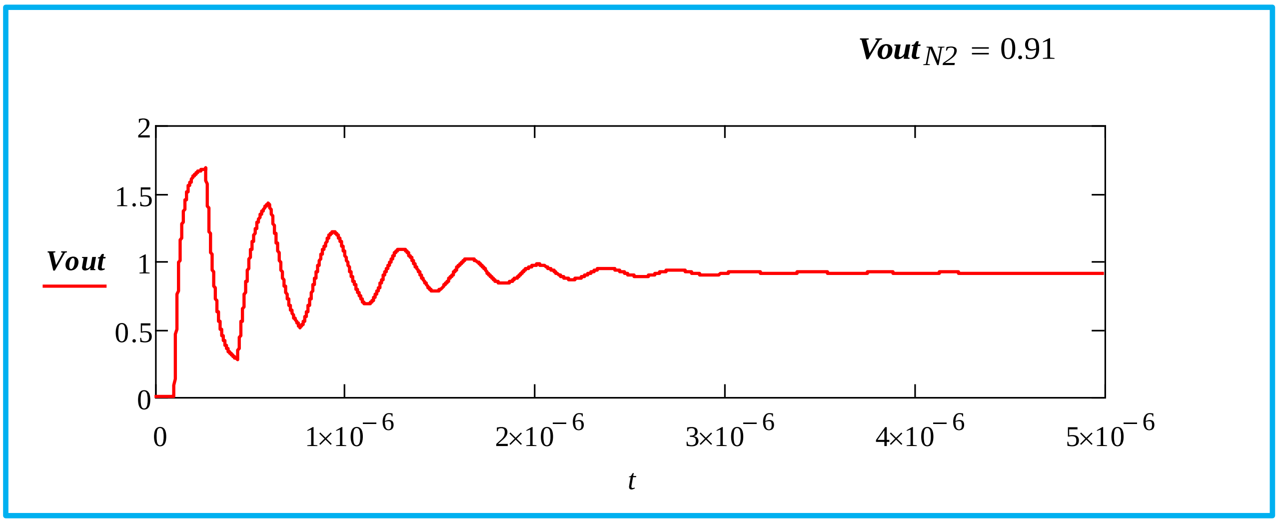


Figure 9 Waveform of the differential-mode voltage at the far end terminals

Conclusion

Cross-coupling is caused by imbalance in the antenna-mode flow of current. This flows along both conductors and emits photons which depart into the environment at the velocity of light. The amplitude of current in one conductor is greater than that in the other. The conductor with the highest current acts as a transmitter, the other as a receiver. The role of each conductor changes after each reflection of the leading edge of the waveform at the open-circuit termination. This imbalance is rapidly damped down by energy absorbed by the resistance at the near end of the line.

Invoking classical theory of transients in inductors and capacitors, a definitive relationship is established between photons and charges. The rate at which photons are emitted is equal to the energy lost in decelerating charges which arrive radially at the surface of a circular section conductor. The radial flow of charges into a circular section conductor is proportional to the rate at which photons arrive at the surface from the environment.