Updating Circuit Theory: Flow of Partial Currents

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Introduction

The analysis of the drift velocity of electrons along a conductor conjures up a picture of water sluggishly flowing along a ditch in the countryside. This is misleading. A more appropriate visualisation of the flow of charges along a conductor is that of an avalanche hurtling down a mountainside.

A reprise of a lesson copied from a blackboard sixty years ago derives a figure for the drift velocity when a copper conductor is carrying its rated current. This was described as a snail's pace. The simulation of the flow of partial currents along a transmission line provides a dramatically different scenario.

If an electron is ejected from a copper atom, that atom becomes positively charged. If an electron is attached, the charge on the atom goes negative. If photons happen to illuminate a small patch on the surface of a copper conductor, then this will initiate a chain reaction. Positive charges will depart along the wire in one direction with negative charges flowing in the other direction.

If a step voltage is applied at the near end of a twin conductor cable, then a wavefront will propagate along that cable, with positive charges flowing along one conductor and negative charges keeping pace along the other conductor.

With a short-circuited termination at the far end the chain reactions continue unabated. A negative wavefront arriving from the return conductor reappears as a positive wavefront departing along the send conductor. A positive wavefront arriving from the send conductor reappears as a negative wavefront departing along the return conductor.

The total current at any point in a conductor is the sum of the partial currents.

Drift velocity

The atomic mass of any element is usually quoted in terms of the unified atomic mass and referred to as the Dalton, which has the symbol '*Da*'

$$Da = 1.6605 \times 10^{-27} \text{ kilogram} \tag{1}$$

The mass of the copper atom, *m*, has been measured to be

$$mass = 63.546 \cdot Da \quad \text{kilogram} \tag{2}$$

The density of copper is

$$density = 8930 \text{ kilogram/m}^3 \tag{3}$$

Hence, the number of atoms of copper per cubic metre is

$$n = \frac{density}{mass} = 8.466 \times 10^{28} \quad \text{atoms / m}^3 \tag{4}$$

The atomic number of copper is 29. That is, there are 29 electrons in orbit round a nucleus which has 29 protons and 35 neutrons. If it is assumed that there is one free electron per atom, then *n* is the number of free electrons per cubic metre capable of drift through the atomic structure. The charge, *e*, on each electron is

$$e = 1.602 \times 10^{-19} \ \text{coulomb} \tag{5}$$

The charge contained by one cubic metre of copper is.

$$Qdensity = n \cdot e \quad \text{coulomb/m}^3 \tag{6}$$

If *I* amperes flows into a conductor for *dt* seconds, then the magnitude of the charge delivered to the conductor will be:

$$Q = I \cdot dt \quad \text{coulomb} \tag{7}$$

If the cross-sectional area of the conductor is *A* and the length of the conductor into which the charge flows is *dx*, then the volume of the conducting material holding this charge will be

$$Volume = A \cdot dx \quad \text{m}^3 \tag{8}$$

The charge density in this volume is

$$Qdensity = \frac{Q}{Volume} = \frac{I \cdot dt}{A \cdot dx} \quad \text{coulomb/m}^3 \tag{9}$$

Using (6) to substitute for *Qdensity*

$$n \cdot e = \frac{I}{A \cdot vel} \quad \text{coulomb/m}^3 \tag{10}$$

Where *vel* is the drift velocity of the electrons.

The diameter *dia* of an AWG 20 conductor is

$$dia = 0.8125 \times 10^{-3} \quad \text{m} \tag{11}$$

So the cross-sectional area is

$$A = \frac{\pi \cdot dia^2}{4} = 5.185 \times 10^{-7} \quad \text{m}^2 \tag{12}$$

The rated current *Irated* is a function of the way the conductor is used. Typically:

$$Irated = 11 \text{ A} \tag{13}$$

Rearranging (10) and assigning values to the parameters *n*, *e*, *A*, and *Irated*

$$vel = \frac{Irated}{A \cdot n \cdot e} = 1.565 \times 10^{-3} \text{ metre/second} \tag{14}$$

This drift velocity has been described as a snail's pace. However, the main point is that there is a superabundance of charge-carrying entities; 8×10^{10} in a cubic micro-metre.

Loop Resistance

The resistivity of copper is

$$\rho = 1.7 \times 10^{-8} \ \Omega \text{ m} \tag{15}$$

If an AWG 20 conductor is used in a 15 metre length of twin-conductor cable, the loop resistance would be

$$R = \frac{\rho \cdot 15 \cdot 2}{A} = 0.984 \ \Omega \tag{16}$$

Transmission Line Model

Figure 1 illustrates a configuration where the resistance at the near end is *Rn* and the resistance at the far end is zero. At the near end, the source voltage is *Vgen*, the incident current is *Ini*, the absorbed current is *Ina*, and the reflected current is *Inr*. At the far end, the incident current is *Ifi*, the absorbed current is *Ifa*, and the reflected current is *Ifr*.

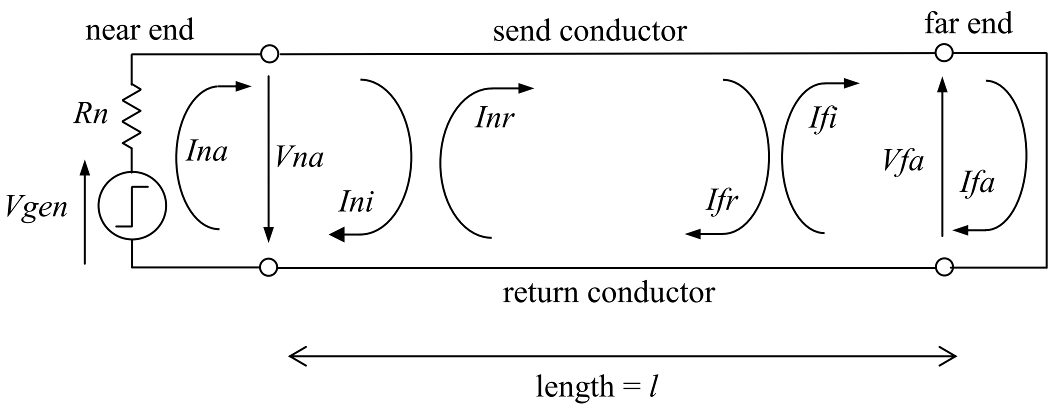

Figure 1 Transmission Line Model

All the currents are defined as loop currents. Positive current is defined as a clockwise flow. This means that positive current flows forward along the send conductor and back along the return conductor. Since *Ini*, *Inr*, *Ifi* and *Ifr* are partial currents, there are two separate currents flowing along each conductor. Since *Ina* is defined as the sum of *Ini* and *Inr*, then the current flow at the near end can be treated as unidirectional. The same is true for *Ifa* at the far end.

The modelling technique assumes that the line is lossless. However, by setting the resistor at the near end equal in value to the series resistance of the conductors, copper loses can be included in the simulation.

Simulation

The loop resistance of a 15 metre length of 20 AWG cable has been calculated to be 0.984 ohm. So, if the terminals at the far end are shorted together and an ohmmeter connected to the terminals at the near end, then the ohmmeter reading would be about 1 ohm. So it is reasonable to assign this value to *Rn*. Assigning a value of 10 V to the generator *Vgen* and a value of 100 ohm to the characteristic resistance *Ro* creates a model which can be used to simulate the responses of the partial currents under steady-state conditions.

Figure 2 is a reproduction of the Mathcad Worksheet used to do just that. Since examples have already been provided of the processing details in previous articles, it is only necessary to state that the vectors ***Ifi***, ***Ifr*** and ***Ifa*** hold values of these parameters over the range zero to *Tsweep*.

Figure 3 is a graph which simulates the response of these three currents over a period of 200 ns. The current at the far end, *Ifa*, is shown as the solid green curve. The incident current, *Ifi*, is shown as the dotted red curve and the reflected current, *Ifr*, is shown as a dotted blue curve. The reflected current is shown as a negative current because it is flowing backwards.

Mechanism

The graph of Figure 3 and an assessment of the computations involved in its creation provide an insight into the mechanism involved in the actual system.

Since a step voltage of 10 V is applied and the characteristic resistance of the line is 100 ohm, the current *Inr* delivered to the line will be 0.1 A. This manifests itself as a series of tiny charges which propagate forward at velocity *v*. In the model, a shift register is used to simulate this action. The number of stages *n* in the register is 100. So each section represents a time duration *dt* of 500 pico-seconds. So the charge delivered is $Inr \cdot dt = 50\,\text{pF}$. During the next 50 ns this charge will propagate forward along the length of the line.

It takes *T* seconds for the leading edge to arrive at the terminals at the far end. During this time, nothing happens at the far end. Then, at $t = 50$ ns the electromagnetic field manifests itself as a voltage of 10 V applied via a resistance of 100 ohm. Hence, the current *Ini* changes from zero to 0.1 A. Then it stays constant at this value for the next 100 ns.

Since the load *Rf* is zero, the current *Ifa* at the far end is twice the value of *Ini*. Hence the reflected current *Ifr* is equal in value to *Ifi*.

$$Vg := 10 \qquad Ro := 100 \qquad Rn := 1 \qquad Rf := 0$$

$$l := 15 \qquad v := 3\cdot10^{8} \qquad T := \frac{l}{v} = 5\times10^{-8}$$

$$n := 100 \qquad dt := \frac{T}{n} = 5\times10^{-10}$$

$$near(Ini, Vg) := \left| \begin{array}{l} Ina \leftarrow \dfrac{2\cdot Ro\cdot Ini + Vg}{Ro + Rn} \\ Inr \leftarrow Ina - Ini \\ (Inr \quad Ina) \end{array} \right. \qquad far(Ifi) := \left| \begin{array}{l} Ifa \leftarrow \dfrac{2\cdot Ro\cdot Ifi}{Rf + Ro} \\ Ifr \leftarrow Ifa - Ifi \\ (Ifr \quad Ifa) \end{array} \right.$$

$$forward(\mathbf{F}, In) := \left| \begin{array}{l} \mathbf{F}_1 \leftarrow In \\ for \;\; x \in n+1 \,..\, 1 \\ \quad \mathbf{F}_{x+1} \leftarrow \mathbf{F}_x \\ If \leftarrow \mathbf{F}_{n+2} \\ (\mathbf{F} \quad If) \end{array} \right. \qquad back(\mathbf{B}, If) := \left| \begin{array}{l} \mathbf{B}_{n+1} \leftarrow If \\ for \;\; x \in 2 \,..\, n+1 \\ \quad \mathbf{B}_{x-1} \leftarrow \mathbf{B}_x \\ In \leftarrow \mathbf{B}_1 \\ (\mathbf{B} \quad In) \end{array} \right.$$

$$N := 4\cdot n \qquad i := 1 \,..\, N \qquad t_i := i\cdot dt \qquad Tsweep := N\cdot dt = 2\times10^{-7}$$

$$(\mathbf{Ifi} \quad \mathbf{Ifr} \quad \mathbf{Ifa}) := \left| \begin{array}{l} \mathbf{F}_{n+2} \leftarrow 0 \\ \mathbf{B}_{n+1} \leftarrow 0 \\ for \;\; i \in 1 \,..\, N-1 \\ \quad \left| \begin{array}{l} Vgen \leftarrow Vg \;\; if \;\; i > 1 \\ (Inr \quad Ina) \leftarrow near(Ini, Vgen) \\ (\mathbf{F} \quad Ifi) \leftarrow forward(\mathbf{F}, Inr) \\ (Ifr \quad Ifa) \leftarrow far(Ifi) \\ (\mathbf{B} \quad Ini) \leftarrow back(\mathbf{B}, Ifr) \\ \mathbf{Ifi}_i \leftarrow Ifi \\ \mathbf{Ifr}_i \leftarrow Ifr \\ \mathbf{Ifa}_i \leftarrow Ifa \end{array} \right. \\ (\mathbf{Ifi} \quad \mathbf{Ifr} \quad \mathbf{Ifa}) \end{array} \right.$$

$$L := T\cdot Ro = 5\times10^{-6} \qquad \mathbf{I}_i := \frac{Vg}{Rn}\left(1 - e^{\frac{-Rn}{L}\cdot t_i}\right)$$

Figure 2 Mathcad worksheet which analyses the response of a short-circuited line

At the far end terminals this reflected current manifests itself as a series of 50 pF discharges which then propagate back towards to near end at velocity *v*. Figure 4 illustrates the responses over a period of 20 micro-second. As expected, the current delivered to the far end rises exponentially towards 10 A.

However, the fact that the partial currents, *Ifi* and *Ifr*, only reach a value of 5 A under steady-state conditions need some explanation.

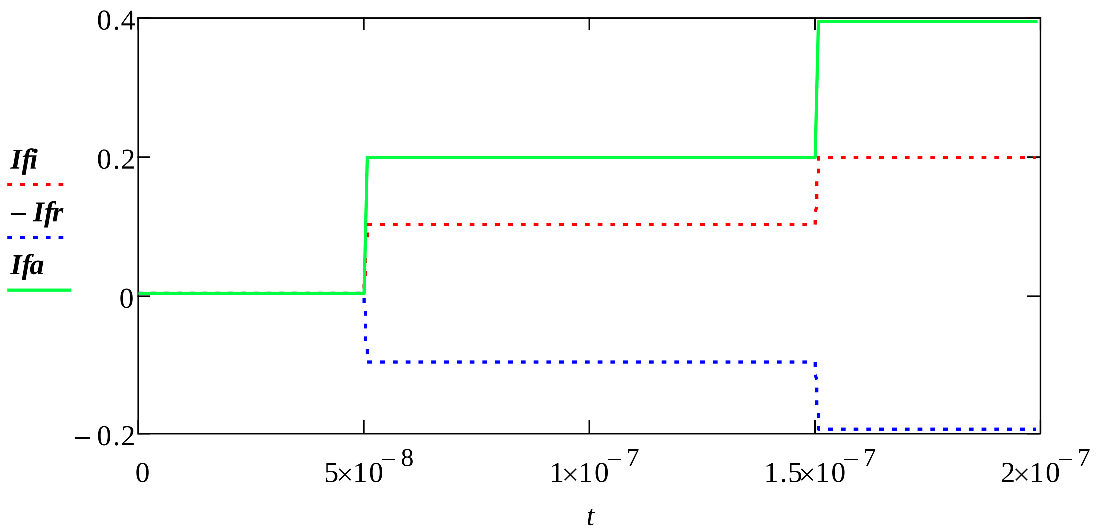

Figure 3 Response of a short-circuited line over a period of four traverses of the step

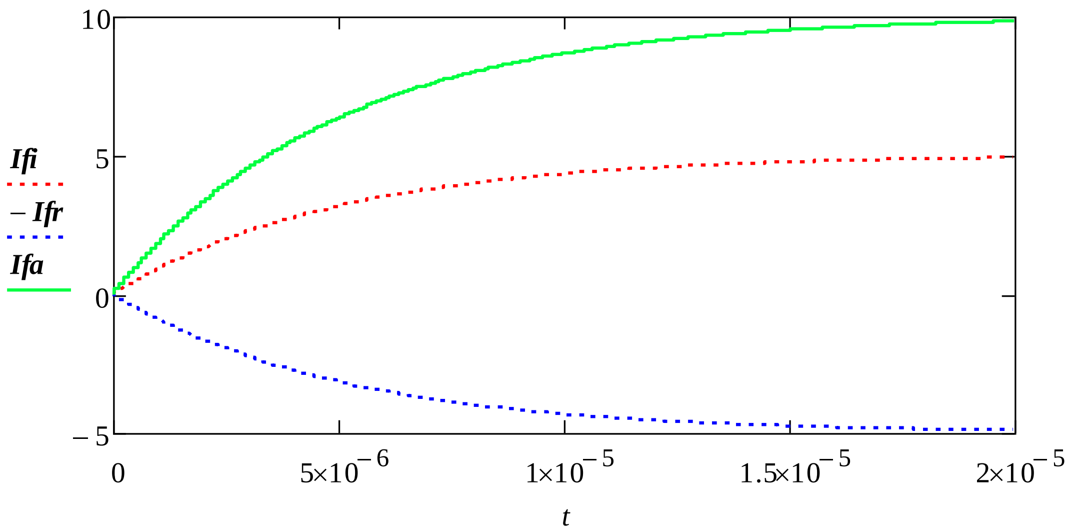

Figure 4 Response of a short-circuited line over a period of 20 micro-seconds

Partial Currents

The relationship between partial current and total current is best explained by analysing the current distribution in the cable at a particular instant. This is illustrated by Figure 5. The worksheet used to create this figure is a slight modification of that recorded in ‘Updating Circuit Theory: Charges and Photons’.

It is assumed that a snapshot is taken of the distribution of current 60 nanoseconds after the step voltage is applied at the near end. That is, 10 nanoseconds after the step has been reflected at the far end. At this instant, the leading edge is propagating back towards the near end, but the current *Ifi* still arriving at the far end is 0.1 A. This is the incident current. The step of 0.1 A propagating back is the reflected current *Ifr*.

This reflected current is really the current in the return conductor continuing its journey back: along the send conductor. The incoming charges at the do not ‘know’ that they have been routed through an angle of 180 degrees at the far end. A negative current flowing in the opposite direction is a positive current.

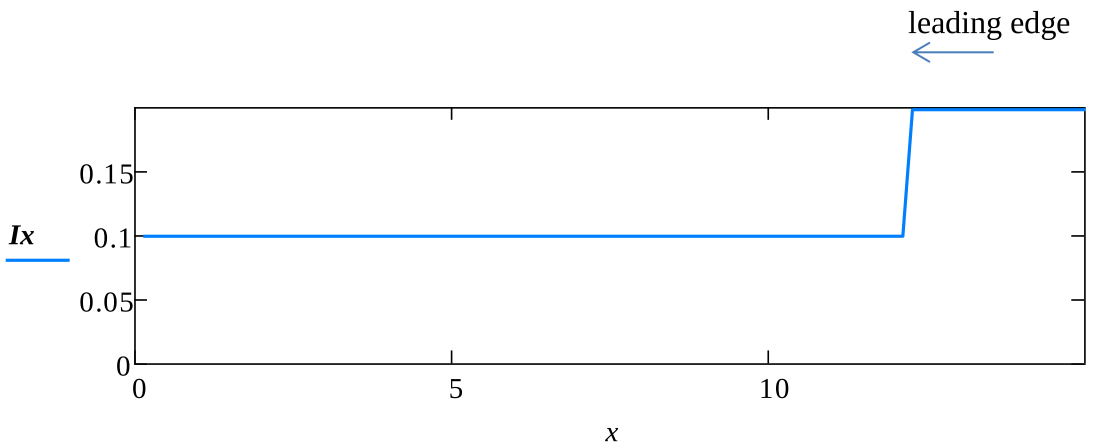

Figure 5 Snapshot of the current distribution along the send conductor at 60 ns

Charge Carriers

Given an insight into the relationship between the propagation of charges and current, it becomes possible to relate these parameters to electrons and atoms.

The analysis of drift velocity has shown that there is a superabundance of charge carriers in the assembly.

If an electron is dislodged from a copper atom, then that atom will become positively charged. It will attract an electron from a neighbouring atom. The neighbouring atom will become positively charged.

Similarly, the dislodged electron will attach itself to a neighbour on the opposite side, causing that neighbour to become negatively charged.

If photons were to illuminate an area on the surface of a copper wire, then many electrons would be displaced from the region at that surface. This would initiate a chain reaction with positive charges propagating in one direction along the conductor and negative charges flowing away in the opposite direction.

Since the cable is short-circuited at the far end, a chain reaction with a positive leading edge moving forward along the send conductor will continue unchecked through the terminals and continue to propagate back along the return conductor with a negative leading edge.

If there is also a chain reaction moving forward along the return conductor with a negative leading edge, then this will also continue through the shorted terminals and propagate back along the send conductor as a step with a positive leading edge.

There is no conflict between the two chain reactions. At any point in the conductor, they support each other. Figure 5 Illustrates this.

Photon action

Such a depiction of charges propagating back and forth along the conductors is only half the picture. The real source of energy is the flow of photons. These propagate to and fro between the conductors and create charges on the surface of both conductors; positive charges on the send conductor and negative charges on the return conductor. It is these induced charges which sustain the current flow.

Since this interchange of photons takes place at the surface of the conductors, the flow of charges along the conductors will be concentrated at the surfaces.

A step voltage applied at the near end of the cable will cause a step current to propagate along each conductor. There will be positive charges at the front end of the send conductor and negative charges at the front end of the return conductor.

As described above, the flow of these charges will continue unchecked at the far end, and the leading edge will propagate back to the near end.

During the transit of energy back down the line, the return conductor acts as a transmitting antenna, delivering energy back to the send conductor. During this time, the send conductor continues to deliver energy to the return conductor. That is, each conductor acts as a transmitting antenna and as a receiving antenna. Both conductors contribute to the kinetic energy being stored in the cable.

Resistance and Inductance

Figure 3 provides a picture of the current increasing as a series of discrete current steps. If the source resistance *Rn* in the model is set at zero, then the amplitude of every step would be 0.1 A and the current would increase linearly with time without limit.

As the current flowing in the each conductor increases, more and more atoms become involved in the propagation of charge. So, more and more of them are forced to flow below the surface of each conductor. Eventually, the movement of charges along the cable involves the entire cross section.

When the conductor is acting as a transmitter, there is radial flow of sub-atomic particles up to the surface, where the charges are trapped and photons are emitted. When the conductor is acting as a receiver, the arriving photons cause similar particles to be driven down onto the conducting material.

There will be impacts between particles moving in opposite directions, whether they are moving laterally, radially, or circumferentially. Particles moving in opposite directions are bound to cause friction. This causes heating. In circuit analysis, this effect is defined by the voltage developed across a resistance.

This resistance causes the amplitude of the steps to decrease with time. Eventually the steps become imperceptible. But that does not mean that the propagation to and fro of the photons ceases. Under steady-state conditions this propagation manifests itself as the magnetic field.

The stored energy in the field is the same as that of the dynamic energy of the charges. An analogy would be that of a flywheel which has picked up speed due to tiny impulses at every rotation.

Electromagnetic Theory uses the concept of inductance to relate the energy in the magnetic field to the voltage developed by a step change in current.

Simple Model

Since the far end is short-circuited, the line behaves as an inductor, where

$$L = T \cdot Ro = 5\times10^{-8} \cdot 100 = 5\times10^{-6} \quad \mathrm{H} \tag{17}$$

The series resistance of the conducting loop has been assumed to be 1 ohm.

If a 10 V step were to be applied to a series L-R circuit, the model used to analyse the response would be as shown by Figure 6.

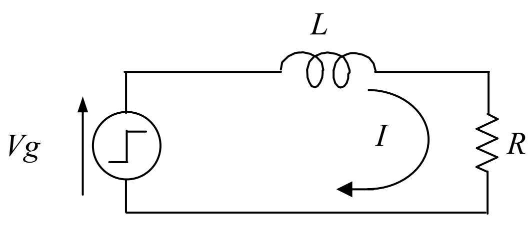

Figure 6 Lumped-parameter model of configuration of Figure 1

The loop equation for transient analysis is

$$Vg = R \cdot I + L \cdot \frac{dI}{dt} \tag{18}$$

Analysing this using Heaviside Transformations gives the relationship between current and time:

$$I = \frac{Vg}{R} \cdot \left(1 - e^{-\frac{R}{L} \cdot t}\right) \tag{19}$$

Plotting this relationship on the same graph as *Ifa* leads to Figure 7. The response *Ifa* at the end terminals of the transmission is illustrated by the solid green curve. The response *I* of the circuit model of Figure 6 is shown as a dotted black curve. The fact that they are identical shows that the modelling technique invoked in the Mathcad Worksheet of Figure 2 produces an accurate simulation. Since the modelling technique is based on the relationships of Transmission Line Theory, the simulation of the flow of partial currents must also be accurate.

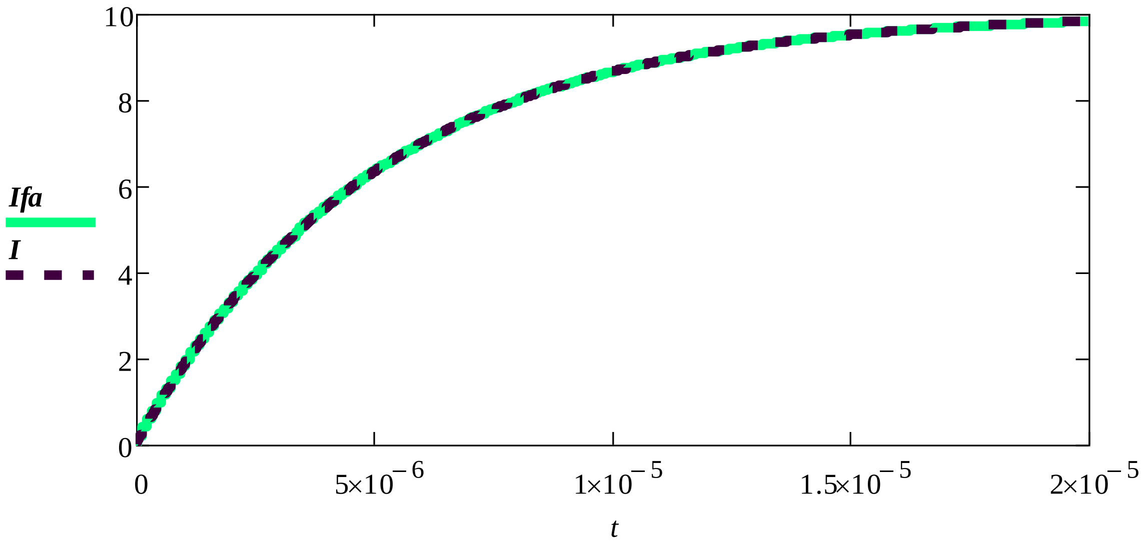

Figure 7 Waveforms, as analysed using Transmission Line equations and Heaviside equations

The description of the mechanisms involved in relating the movement of electrons and the action of photons is consistent with both the simulation of Figure 4 and with the known properties of photons and electrons.

The properties of the **photon** are defined by Wikipedia as: '*The photon is a type of elementary particle. It is the quantum of the electromagnetic field including electromagnetic radiation such as light and radio waves, and the force carrier for the electromagnetic force. Photons are massless, and they always move at the speed of light in vacuum*.' It has also been stated by the scientific community that photons do not carry charge.

The properties of atoms and electrons can be found in most science data books.

Conclusion

A plausible description has been provided of the mechanism underlying the behavior of photons, electrons and atoms in a transmission line. It provides a solid basis for further analysis as well as an improved understanding of the phenomenon.

Comments are welcome.