Updating Circuit Theory: Transient Emission

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Introduction

An experiment is described which enables transient radiation to be measured and analysed. It involves the application of a step pulse to one end a twin-conductor cable which is open at the far end and the use of a current transformer to monitor the differential-mode current. The waveform of this current is initially that of a square wave, but it quickly morphs into a sine wave. A circuit model of this configuration is created to replicate the observed waveform. Analysis of the model provides an insight into the mechanisms involved.

Setup

An Interface Box was first constructed in order to apply a step pulse to such a cable whilst maintaining isolation between the signal generator and the input terminals. This included a common-mode choke which provided high impedance to any antenna-mode current flow between the assembly under test and the ground terminals of the generator. The use of a current transformer to monitor the current ensured that there was complete isolation between the oscilloscope and the cable. Included in the Interface Box was a simple resistor network which provided a 50 ohm load to the Signal Generator and a low resistance signal source at the input to the cable.

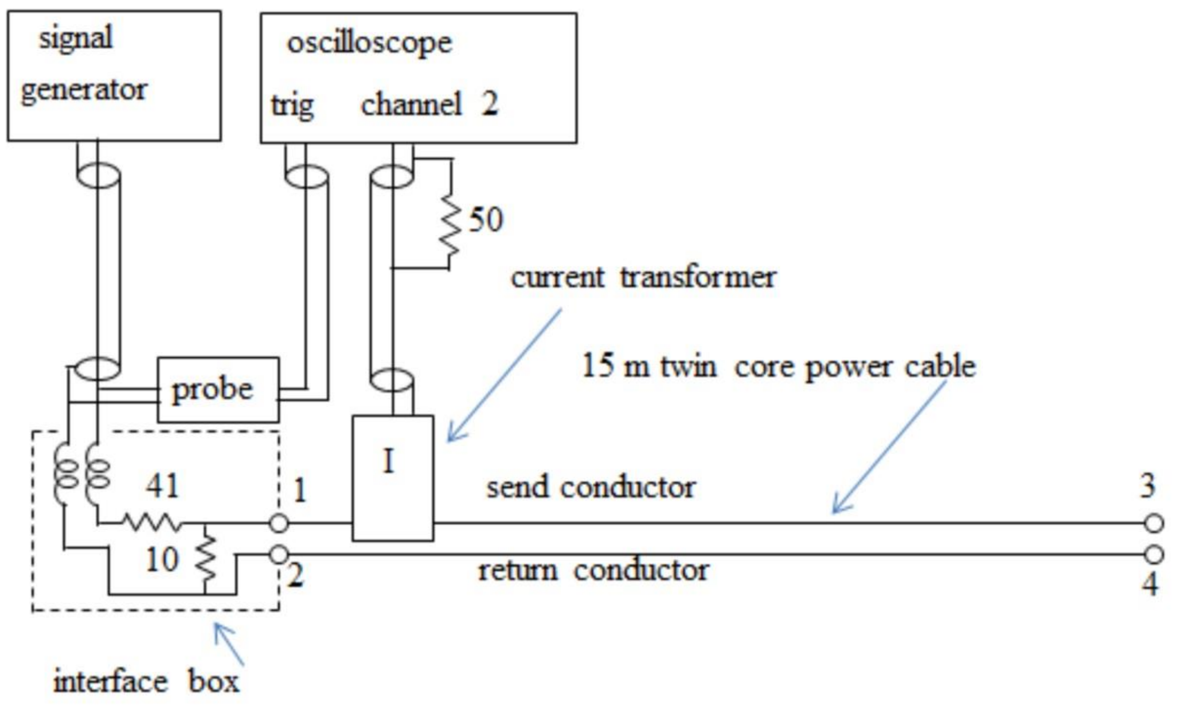


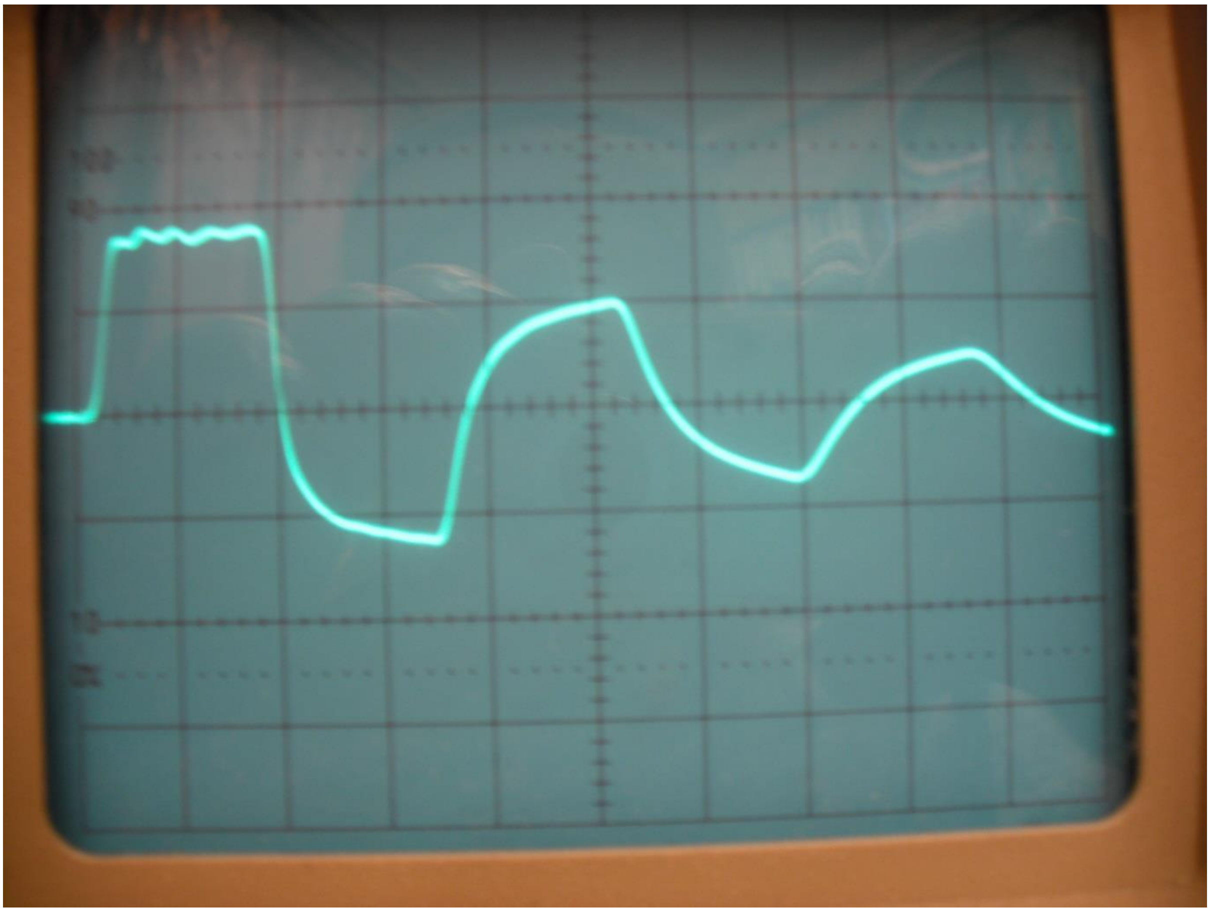
Figure 1 Setup used to investigate the transient response of twin-conductor cable.

Test Method

Since the cable was open-circuit at the far end and supplied via a low resistance at the near end, reflections would occur at both terminations. The response to a step voltage input was expected to be a ringing pulse which decayed relatively slowly. So the Signal Generator was set to provide a square wave pulse at 100 KHz. This gave a mark time of 20 microseconds and a mark/space ratio of unity. This delay provided more than enough time for the reverberations of the line to die down before the occurrence of the next step.

The current transformer was a clamp-on device allowed the voltage waveform on channel 2 of the oscilloscope to replicate the waveform of the current in the send conductor. A times-ten voltage probe was used to provide a trigger pulse to the oscilloscope, to eliminate any jitter in the monitored waveform.

A photograph of the current waveform is reproduced by Figure 2.



Vertical scale: 10 mV/div   Horizontal scale 100 ns/div

Figure 2 Waveform of current in the send conductor

Vertical Scaling

The objective now was to create a circuit model which replicated the waveform of Figure 2. This meant that the response of the test equipment needed to be simulated as well as that of the cable. So the first task was to measure the amplitude of the voltage delivered to the input terminals (terminals 1 and 2 of Figure1).

In Figure 1, the 41 ohm resistor is in series with the 50 ohm output resistor of the signal generator. So the source resistance of the supply to the input terminals was:

$$Rg = \frac{10 \cdot (41 + 50)}{10 + 41 + 50} = 9.01 \text{ ohm} \tag{1}$$

The relationship between Figures 3a and 3b illustrate this. The amplitude of the voltage step *Vin*, as monitored by the voltage probe, was

$$Vin = 0.84 \text{ Volt} \tag{2}$$

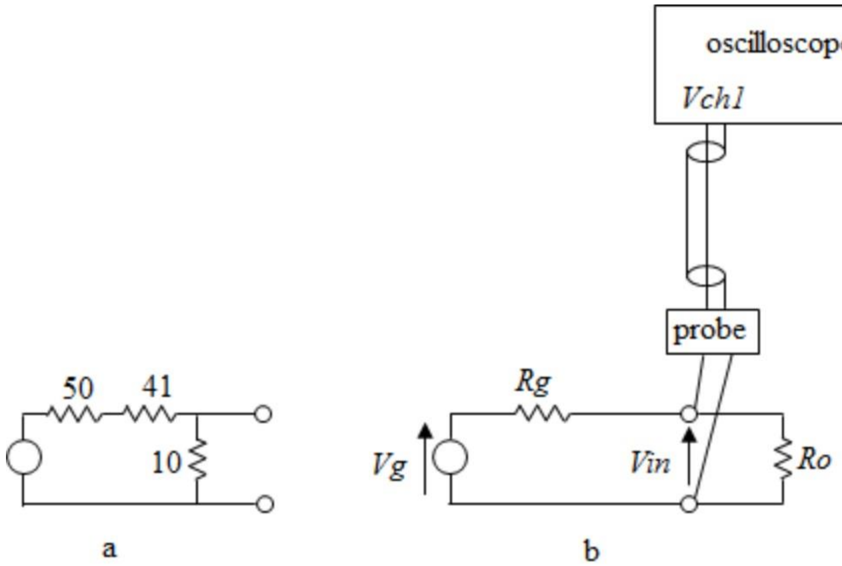
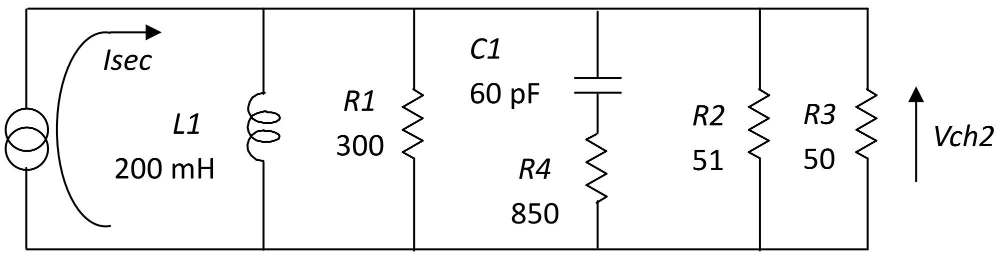


Figure 3 Circuit model of input circuitry
a model used to derive source resistance
b relating voltages *Vch1, Vin and Vg*

Calibration of the current probe in the frequency domain enabled the circuit model of Figure 4 to be created. This related the current in the primary winding, *Iprim*, to the amplitude voltage *Vch2* monitored at the oscilloscope, over the frequency range 50 kHz to 20 Mhz. The number of turns on the secondary winding was

$$Turns = 10 \tag{3}$$



$$Iprim = Isec \cdot Turns$$

Figure 4 Circuit model of the current transformer

A full description of the calibration process is provided by the article ‘Updating Circuit Theory: The Current Monitor Transformer’. For transient analysis, it can be assumed that the inductor *L1* is open-circuit and the capacitor *C1* is short circuit. So the ratio between *Isec* and *Vch2* is

$$\frac{Isec}{Vch2} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4} = 0.044 \tag{4}$$

And the transfer resistance *RT* is

$$RT = \frac{Vch2}{Iprim} = \frac{Vch2}{Isec \cdot Turns} = 2.27 \text{ ohm} \tag{5}$$

The amplitude *Vch2* of the first step in the waveform monitored on channel 2 of the oscilloscope was derived from examination of the waveform of Figure 2.

$$Vch2 = 17\,\text{mV} \tag{6}$$

This means that the measured value of the differential-mode current flowing in the cable changes during the first transit was:

$$Idiff = \frac{Vch2}{RT} = 7.489 \text{ mA} \tag{7}$$

So, the characteristic resistance *Ro* of the cable was

$$Ro = \frac{Vin}{Idiff} = 112 \text{ ohm} \tag{8}$$

From Figure 3b

$$Vg = Vin \cdot \frac{Rg + Ro}{Ro} = 0.907 \tag{9}$$

Horizontal Scaling

Each horizontal division of the graticule shown on Figure 2 represents 100 ns. If the vertical line on the left hand side of the display represents zero time, then the time *T1* of the first rising edge of the waveform is

$$T1 = 20 \text{ ns} \tag{10}$$

In simulating the response, *T1* is the time that the input voltage *Vgen* switches from zero to *Vg*.

The first trailing edge occurs at 185 ns. The interval between the rising and falling edges represents the time taken for the leading edge to propagate from the near end of the cable to the far end and then back to the near end. If *T* is the time for one traverse, then

$$T = \frac{185 - 20}{2} = 82.5 \text{ ns} \tag{11}$$

The time taken for the scan to sweep from the first to the tenth graticule line is

$$T2 = 1000 \text{ ns} \tag{12}$$

Modelling

Initial observation was that it was not a square waveform as would have been predicted using the textbook analysis of Electromagnetic Theory. Even so, the response for the first 165 ns did approximate that of a square wave. The finite rise time was an indication of the response time of the oscilloscope. The small ripple at the top of the waveform was probably due to reflections at the current transformer. Textbook analysis indicates that the time duration of 165 ns represents the time for the leading edge to propagate along the line, be reflected, and propagate back to the near end.

The first departure from the textbook response was the exponential decay of the first trailing edge. This indicated that current was departing from the line via capacitive coupling. The waveform then gradually morphed from a square wave to a sine wave, as well as decaying in amplitude. This decay was to be expected, since energy was departing from the line in the form of electromagnetic radiation. Losses on the source resistance would also contribute to this decay.

Initial attempts to model this response were reasonably successful, but the simulation failed to replicate the kinks at the top of the second and third peaks. It was then reasoned that the departure of current occurred during the time the pulse was propagating forward from the near end to the far end. The send conductor was acting as a transmitting antenna. But during the time the current was flowing back to the near end, the return conductor was acting a transmitting antenna and delivering energy back to the send conductor.

The computations follow the same sequence as that described in the article ‘Updating Circuit Theory: Power Line Filter’. But this time the focus of attention is on the behaviour of the cable, rather than on that of the terminations.

Figure 5 illustrates the sequence of calculations. The parameters are:

Ina is the amplitude of the current absorbed by the interface at the near end,
Inr is the amplitude of the current reflected back into the line at the near end,
Ind is the same amplitude as *Inr*, but delayed by *T* seconds,
Ins is the current needed to store charge *Qns* at the surface of the send conductor,
Ifi is the current delivered to the far end; the incident current,
Ifa is the current absorbed at the far end,
Ifr is the current reflected back into the line at the far end,
Ifd is the same amplitude as *Ifr*, but delayed by *T* seconds,
Ifs is the current needed to store charge *Qfs* on the surface of the return conductor,
Ini is the incident current arriving at the near end.

It is assumed that the energy involved in storing charge *Qns* on the send conductor departs from the system in the form of an electromagnetic wave.

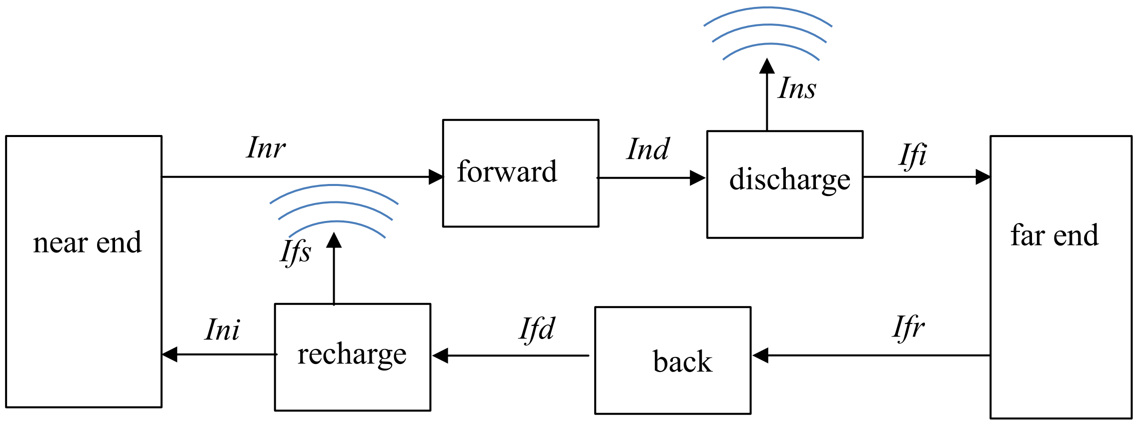


Figure 5. Block diagram which identifies the signal processing functions

Equations

The equations defining the responses at the near and far end are basically the reflection equations derived in any textbook on Electromagnetic Theory. The signal processing involved in simulating the time delays is the same as that used in the article ‘Updating Circuit Theory: Power Line Filter’. Equations involved in the simulation of the discharge current *Ins* can be derived from the circuit models of Figure 6

The energy source in the line can be represented as a current *Ind* flowing in the characteristic resistance *Ro*. The voltage between the conductors is

$$Vnd = Ro \cdot Ind \qquad (13)$$

The current flowing in the capacitor *Crad* can be derived from Figure 6b

$$Vnd = Ro \cdot Ins + \frac{Qns}{Crad} \tag{14}$$

From equations (13) and (14)

$$Ins = Ind - \frac{Qns}{Ro \cdot Crad} \tag{15}$$

Since the current *Ins* has departed from the system, the current *Ifi* arriving at the far end must be

$$Ifi = Ind - Ins \tag{16}$$

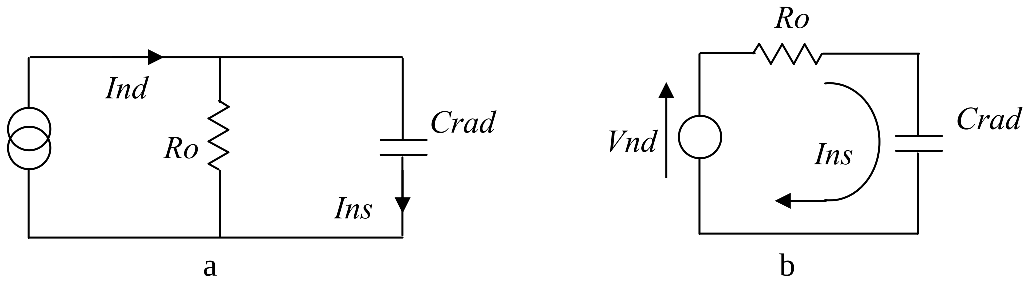


Figure 6 Deriving the amplitude of the discharge current *Ins*
a Using a current source to simulate loop current in the cable
b Converting the circuit model to one which uses a voltage source

The same relationships apply to the derivation of the current *Ifs* which represents the departure of energy from the return conductor. But this energy is not lost to the system. It is picked up by the send conductor, as illustrated by Figure 5.

From Figure 7

$$Ifs = Ifd - \frac{Qfs}{Ro \cdot Cret} \tag{17}$$

and the loop current delivered to the near end is assumed to be:

$$Ini = Ifd + Ifs \tag{18}$$

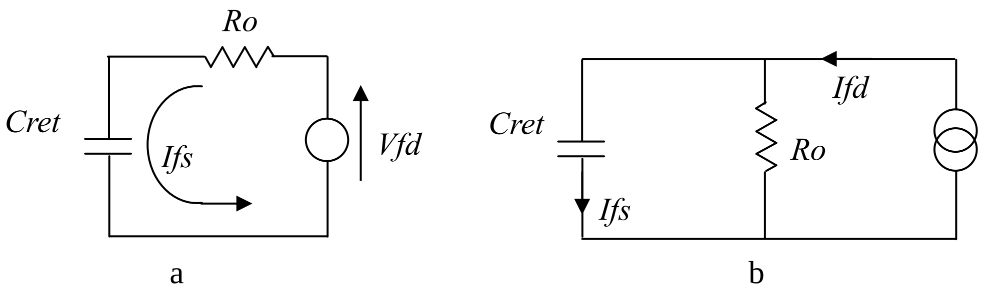


Figure 7 Deriving the amplitude of the recharge current *Ifs*
a Using a current source to simulate loop current in the cable
b Converting the circuit model to one which uses a voltage source

Computation

The computations are set out in a two-page Mathcad Worksheet. The first page is illustrated by Figure 8.

The constants *Vg*. *Ro*, and *Rg* are defined by equations (9) , (8) and (1). To simulate an open circuit termination at the far end, the load *RL* was set at 100 M-ohm. The propagation time *T* is defined by equation (11). It is assumed that the line is divided into $n = 100$ segments. So the time taken for a transient pulse to propagate from one segment to the next is

$$dt = \frac{T}{n} \tag{19}$$

The values of the capacitors *Crad* and *Cret* at the start of the computation were unknown. Initially, arbitrary values were assigned to these components.

The functions *near* ( ), *forward* ( ), *far* ( ), and *back* ( ) are essentially those used in the article 'Updating Circuit Theory: Charges and Photons'

The *discharge* ( ) function is developed from equations (15) and (16). The charge *Qns* accumulated on the capacitor *Crad* after *dt* seconds is

$$Qns \leftarrow Qns + Ins \cdot dt \tag{20}$$

and the incident current *Ifi* arriving at the far end is as defined by equation (16).

The recharge ( ) function is developed from equations (17) and (18). The charge *Qfs* in the capacitor *Cret* is

$$Qfs \leftarrow Qfs + Ifs \cdot dt \tag{21}$$

A copy of the second page of the Mathcad Worksheet is provided by Figure 9

The value of the transfer resistance *RT* is provided by equation (5). This constant relates the amplitude of the voltage observed on channel 2 of the oscilloscope, *Vch2,* to the amplitude of the current in the primary of the current monitor transformer. This is simulated by the current *Ina* flowing in the resistor *Rg* at the near end of the line.

The constant *T1* is defined by equation (10). The constant *N1* defines the number of time steps between the start of the scan and the first leading edge of the waveform. The sweep time *T2* is defined by equation (12) and the constant *N2* is the number of time steps of the simulation.

The main program invokes the functions defined in the block diagram of Figure 5 and creates the vector ***A*** which records the value of the selected parameter at every time ***t***. In this particular case, it is the amplitude of the voltage which would be displayed at channel 2 of the oscilloscope, and is proportional to the current *Ina* flowing in the resistor *Rg* .

$Vg := 0.91 \quad Ro := 112 \quad Rg := 9 \quad RL := 100 \cdot 10^{6}$

$T := 82.5 \cdot 10^{-9} \quad n := 100 \quad dt := \frac{T}{n}$

$Crad := 280 \cdot 10^{-12}$ Radiating capacitor: Select-on-test variable.

$Cret := 100 \cdot 10^{-12}$ Recharging capacitor: Select-on-test variable.

$$near(Ini, Vg) := \begin{vmatrix} Ina \leftarrow \frac{2 \cdot Ro \cdot Ini + Vg}{Ro + Rg} \\ Inr \leftarrow Ina - Ini \\ (Inr \quad Ina) \end{vmatrix}$$

$$forward(\mathbf{F}, Inr) := \begin{vmatrix} \mathbf{F}_1 \leftarrow Inr \\ \text{for } x \in n+1 \, .. \, 1 \\ \quad \mathbf{F}_{x+1} \leftarrow \mathbf{F}_x \\ Ind \leftarrow \mathbf{F}_{n+2} \\ (\mathbf{F} \quad Ind) \end{vmatrix}$$

$$discharge(Ind, Qns) := \begin{vmatrix} Ins \leftarrow Ind - \frac{Qns}{Ro \cdot Crad} \\ Ifi \leftarrow Ind - Ins \\ Qns \leftarrow Qns + Ins \cdot dt \\ (Ifi \quad Ins \quad Qns) \end{vmatrix}$$

$$far(Ifi) := \begin{vmatrix} Ifa \leftarrow \frac{2 \cdot Ro \cdot Ifi}{Ro + RL} \\ Ifr \leftarrow Ifa - Ifi \\ (Ifr \quad Ifa) \end{vmatrix}$$

$$back(\mathbf{B}, Ifr) := \begin{vmatrix} \mathbf{B}_{n+1} \leftarrow Ifr \\ \text{for } x \in 2 \, .. \, n+1 \\ \quad \mathbf{B}_{x-1} \leftarrow \mathbf{B}_x \\ Ifd \leftarrow \mathbf{B}_1 \\ (\mathbf{B} \quad Ifd) \end{vmatrix}$$

$$recharge(Ifd, Qfs) := \begin{vmatrix} Ifs \leftarrow Ifd - \frac{Qfs}{Ro \cdot Cret} \\ Qfs \leftarrow Qfs + Ifs \cdot dt \\ Ini \leftarrow Ifd + Ifs \\ (Ini \quad Ifs \quad Qfs) \end{vmatrix}$$

Figure 8 Defining input variables and functions used in the main program.

$RT := 2.27$ Transfer resistance: Ratio of *Vch2* to current in the line, *Ina*.

$T1 := 20 \cdot 10^{-9}$ $N1 := floor\left(\frac{T1}{dt}\right) = 24$ the count, *i*, at leading edge

$T2 := 1000 \cdot 10^{-9}$ $N2 := floor\left(\frac{T2}{dt}\right) = 1212$ the count, *i*, at end of sweep

$i := 1 .. N2$ $\mathbf{t}_i := i \cdot dt$ the time variable.

$$
\mathbf{Ina} := \left|
\begin{array}{l}
\mathbf{F}_{n+2} \leftarrow 0 \\
\mathbf{B}_{n+1} \leftarrow 0 \\
\text{for } i \in 1 .. N2 \\
\quad \left|
\begin{array}{l}
Vgen \leftarrow Vg \ \text{ if } \ i > N1 \\
(Inr \ \ Ina) \leftarrow near(Ini, Vgen) \\
(\mathbf{F} \ \ Ind) \leftarrow forward(\mathbf{F}, Inr) \\
(Ifi \ \ Ins \ \ Qns) \leftarrow discharge(Ind, Qns) \\
(Ifr \ \ Ifa) \leftarrow far(Ifi) \\
(\mathbf{B} \ \ Ifd) \leftarrow back(\mathbf{B}, Ifr) \\
(Ini \ \ Ifs \ \ Qfs) \leftarrow recharge(Ifd, Qfs) \\
\mathbf{A}_i \leftarrow RT \cdot Ina
\end{array}
\right. \\
\mathbf{A}
\end{array}
\right.
$$

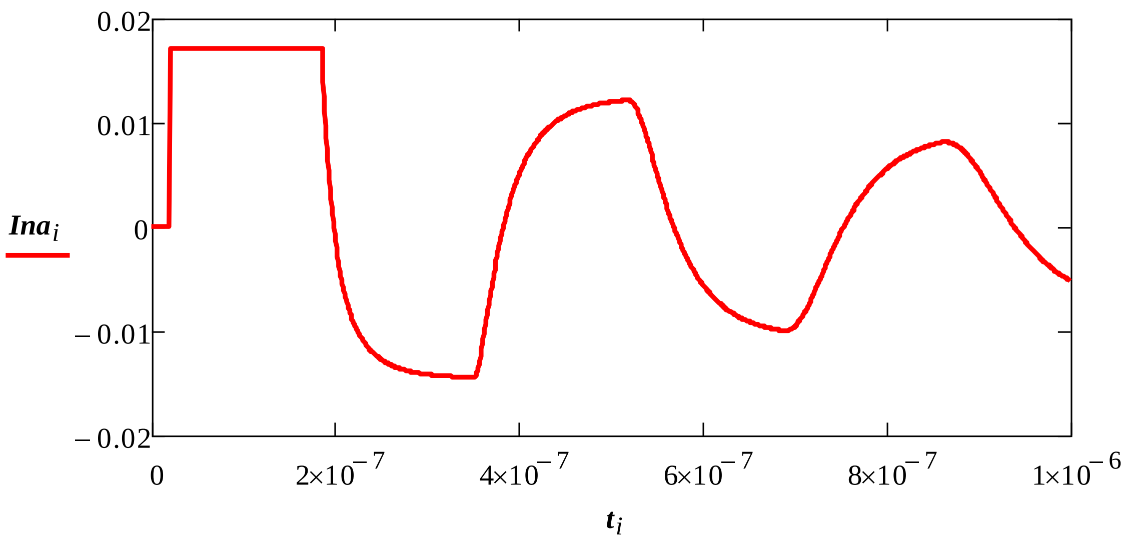


Figure 9 Main program and graph showing the response of the current at the near end

The result of the simulation is displayed on the graph at the bottom of Figure 9. This shows the waveform of the current *Ina* at the near end of the cable. It took about half a dozen iterations of the program to select values for the constants *Crad* and *Cret*.

Assessment

Comparing this graph with the actual waveform recorded on Figure 2 shows that it is a fair representation of the actual response. This being so, it becomes possible to deduce the mechanisms involved in propagating a transient signal along a twin conductor cable. The basic model acts as a perfect transmission line. The inclusion of *Crad* allows the radiation losses to be simulated. The inclusion of *Cret* improves the similarity between the response of the model and the recorded waveform.

It can be reasoned that, during forward propagation of the wave, the send conductor acts as a transmitting antenna whilst the return conductor act as a receiving antenna. During the propagation of reflected energy back from the far end, the functions of the conductors are reversed. The return conductor acts as a transmitter and the send conductor acts as a receiver.

This continuous interchange of energy causes most of that energy to flow along the path defined by the routing of the cable that energy. Since current is also emitted from the outward-facing surfaces of the conductors, that current disappears into the environment and constitutes a loss of energy from the system.

Transient Emission

Figure 10 illustrates the waveform of the current *Ins*. This is a measure of the transient emission from the cable. Since there is a delay of 83 ns between the time the current step is delivered to the near end and the time it arrives at the far end, the waveform of Figure 10 is a record of what has happened in the past. This means that during the first nanosecond, all the current delivered to the send conductor departs into the environment.

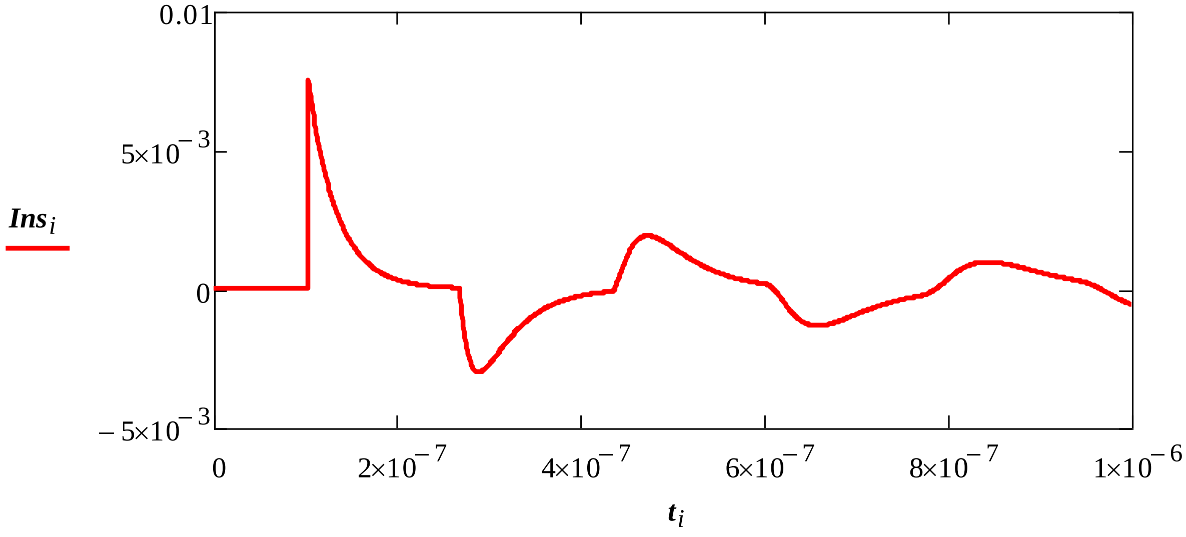


Figure 10 Waveform of the transient emission from the cable

Surface charges

In departing into the environment, the current *Ins* in the send conductor leaves a charge of *Qns*. So the voltage *Vns* on the surface of the send conductor is

$$Vns = \frac{Qns}{Crad} \tag{22}$$

In departing from the send conductor, the current *Ifs* flowing back towards the near end deposits a charge *Qfs*. So the voltage *Vfs* on the surface of the return conductor is:

$$Vfs = \frac{Qfs}{Cret} \tag{23}$$

Figure 11 shows the waveforms of these voltages over a period of 10 micro-seconds. As expected, each voltage settles down to a constant value of 455 mV. The figure also records the applied voltage, *Vg*.

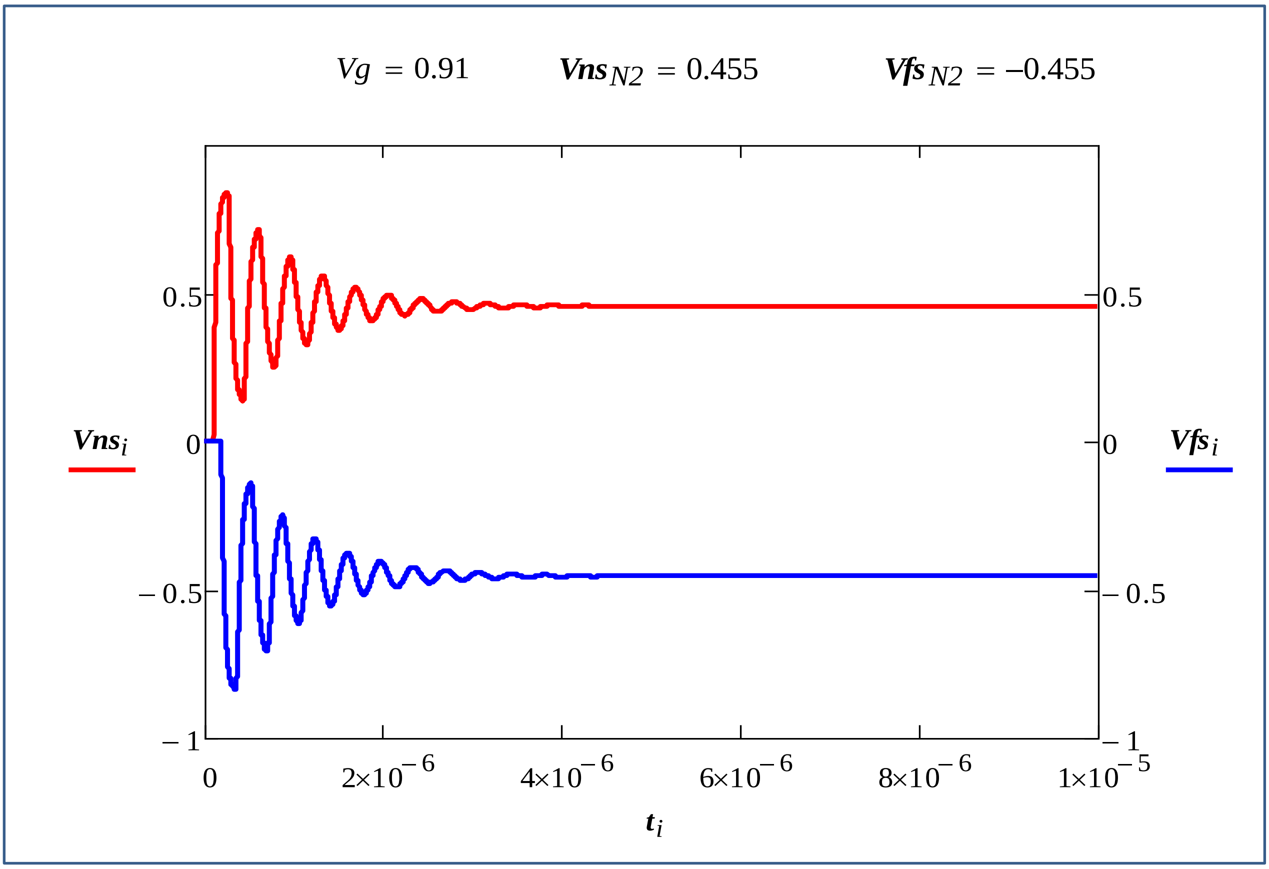


Figure 11 Waveforms of the voltages on the send and return conductors.

Conclusion

This article has shown how a model can be created to simulate the transient emission of a twin-core cable. Since the response is a fair copy of the actual waveform and since the techniques and equations of Circuit Theory have been used exclusively in the creation of the model, it has been demonstrated that the method can be used to analyse the performance of any similar assembly.

That is, the transient emission of any twin conductor cable can be measured using simple test equipment and analysed using general purpose mathematical software.

Assessment of the model also provides an insight into the mechanisms involved in transient emission.