Updating Circuit Theory: Power Line Transients

Ian Darney

Introduction

Transient spikes due to load switching are a prolific source of interference in any electronic system. Circuit models can be created to simulate the interaction between equipment and cable.

Power Line filters are an essential part of the design of any unit of electronic equipment because they form a barrier between the electromagnetic interference (EMI) which has been picked up by the supply line and the equipment to which the power is being delivered. But they do this by reflecting that EMI back down the line. This doubles the level of unwanted power carried by the cable and doubles the EMI created by that line. Moreover, when the equipment unit is switched off, the transient power stored in the filter has only one place to go; out into the environment.

This article analyses the effect of an inductive load and of a capacitive load on the cable. It concludes that a filter with reactive components will interact with the supply cable to create high frequency resonance, and this resonance will be a rich source of EMI.

Capacitive Load

If a switch at the input end of a power line is closed, as illustrated by Figure 1, then a step current will start to flow along a path defined by the routing of the cable. This step will propagate at a velocity similar to that of the speed of light in a vacuum. If it is assumed that the line is lossless, then the amplitude of the step arriving at the far end of the line will be the same as that delivered to the near end.

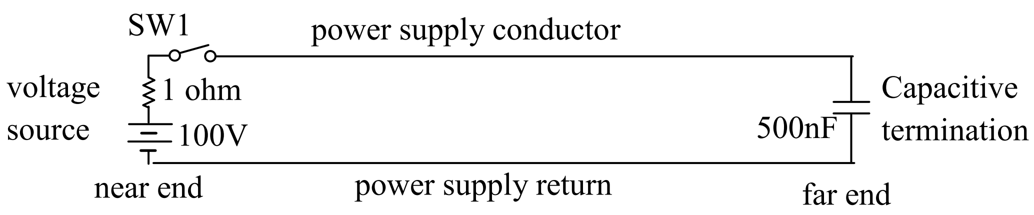

Figure 1 Power line wiring diagram, capacitive termination

The current *In* delivered to the near end is

$$In = \frac{Vn}{Ro} \tag{1}$$

Where *Vn* is the voltage at the near end and *Ro* is the characteristic resistance of the line. If *l* is the length of the line and *v* is the velocity of propagation, then the time *T* taken for the step to traverse from the near end to the far end will be

$$T = \frac{l}{v} \qquad (2)$$

So the current arriving at the far end *If* will be the same as that which existed at the near end *T* seconds previously. By treating the line as a number *n* of segments of equal length, this propagation can be simulated by a shift register. The time taken for the step to move from one segment to the next is

$$dt = \frac{T}{n} \qquad (3)$$

Where dt is a finite period of time.

It is assumed that the generator voltage *Vg* is constant at 100V and that the switch closes at time 0+*dt* and switches off at time *Toff*. The number of time steps *Noff* at that instant is

$$Noff = \frac{Toff}{dt} \qquad (4)$$

The parameters involved in simulating the propagation of energy back and forth are defined in figure 2. The far end equations can be derived from the circuit of Figure 3.

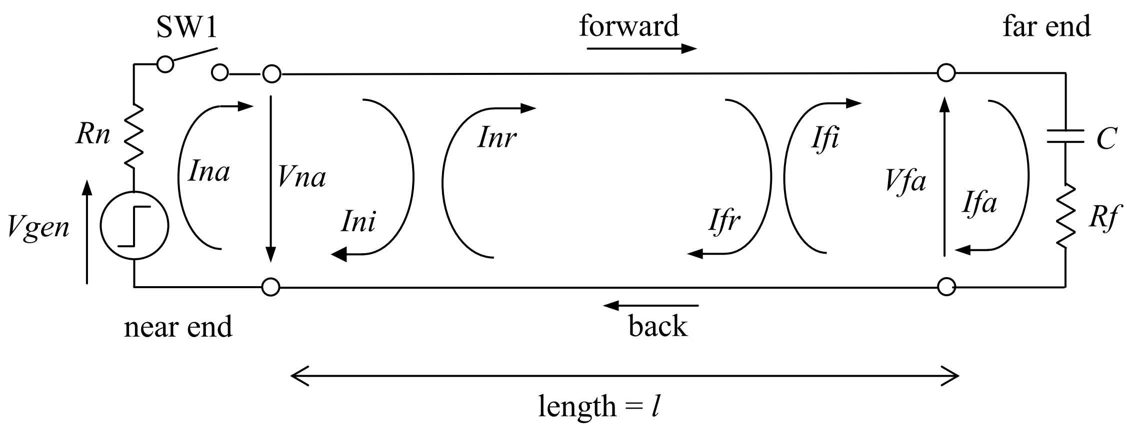

Figure 2 Circuit model of transmission line

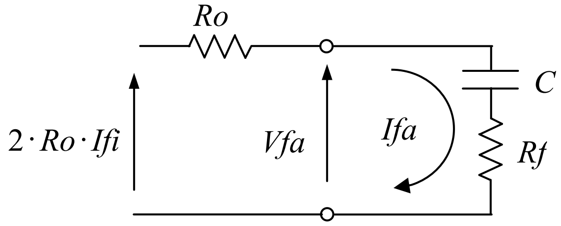

Figure 3 Circuit model used to calculate the response at the far end

From Figure 3

$$2 \cdot Ro \cdot Ifi = \frac{Qfa}{C} + \left(Ro + Rf \right) \cdot Ifa \qquad (5)$$

This leads to the definition of the Mathcad function *far* (*Ifi* , *Qfa*) , shown on Figure 4. This takes the parameters *Ifi* and *Qfa* as input, calculates what the values of *Ifr*, *Ifa* and *Qfa* would be after time *dt* and provides these as the output.

$$far\,(Ifi\,, Qfa) := \left| \begin{array}{l} Ifa \leftarrow \dfrac{1}{Rf + Ro} \cdot \left(2 \cdot Ro \cdot Ifi - \dfrac{Qfa}{C} \right) \\ Qfa \leftarrow Qfa + Ifa \cdot dt \\ Ifr \leftarrow Ifa - Ifi \\ (\,Ifr \quad Ifa \quad Qfa\,) \end{array} \right.$$

Figure 4. Mathcad function which calculates the response at the far end.

Having defined this function, the task of computing the response of the system can be carried out by a Mathcad worksheet similar to that used in the article ‘ Updating Circuit Theory: Charges and Photons’. So it was just a matter of copying that program onto a blank worksheet and altering a few details. The result is the worksheet shown in Figures 5 and 6.

The basic parameters are defined at the top of Figure 5. It is assumed that the source voltage is 100 V, the characteristic resistance of the line is 100 ohm, the length of the line is 15 m, the velocity of propagation is that of light in a vacuum, the source resistance is 1 ohm, and that the capacitor at the far end is 500 nF. For this computation it is assumed that there is no resistance in series with the capacitor. Hence $Rf = 0$.

The time taken for a signal to make one traverse of the line is 50 ns. It is assumed that the line is divided into ten segments. So the calculations are carried out at 5 ns intervals.

The functions defined on this first page of the worksheet calculate the response at the near end of the line, the shifting of the signal from the near to the far end, the response at the far end, and the shifting of the reflected signal back to the near end.

The control parameters for the computation are defined at the top of Figure 6. It is assumed that the switch SW1 closes at 5 ns and that it opens at 50 microseconds. This means that the voltage applied to line drops to zero when the number of iterations *Noff* is ten thousand.

$Vg := 100 \quad Ro := 100 \quad Rn := 1 \quad C := 500 \cdot 10^{-9} \quad Rf := 0$

$l := 15 \quad v := 3 \cdot 10^{8} \quad T := \frac{l}{v} = 5 \times 10^{-8}$

$n := 10 \quad dt := \frac{T}{n} = 5 \times 10^{-9}$

$$near(Ini, Vg) := \left| \begin{array}{l} Ina \leftarrow \dfrac{2 \cdot Ro \cdot Ini + Vg}{Ro + Rn} \\ Inr \leftarrow Ina - Ini \\ (Inr \quad Ina) \end{array} \right.$$

$$forward(\mathbf{F}, In) := \left| \begin{array}{l} \mathbf{F}_1 \leftarrow In \\ \text{for } x \in n+1 .. 1 \\ \quad \mathbf{F}_{x+1} \leftarrow \mathbf{F}_x \\ If \leftarrow \mathbf{F}_{n+2} \\ (\mathbf{F} \quad If) \end{array} \right.$$

$$far(Ifi, Qfa) := \left| \begin{array}{l} Ifa \leftarrow \dfrac{1}{Rf + Ro} \cdot \left(2 \cdot Ro \cdot Ifi - \dfrac{Qfa}{C} \right) \\ Qfa \leftarrow Qfa + Ifa \cdot dt \\ Ifr \leftarrow Ifa - Ifi \\ (Ifr \quad Ifa \quad Qfa) \end{array} \right.$$

$$back(\mathbf{B}, If) := \left| \begin{array}{l} \mathbf{B}_{n+1} \leftarrow If \\ \text{for } x \in 2 .. n+1 \\ \quad \mathbf{B}_{x-1} \leftarrow \mathbf{B}_x \\ In \leftarrow \mathbf{B}_1 \\ (\mathbf{B} \quad In) \end{array} \right.$$

Figure 5 First page of Mathcad worksheet

The main program simply takes the output of each subroutine and supplies these to the input of the next. The value of the current at the near end at each step is recorded in the vector ***Ina*** and this is the final output.

$$Toff := 5 \cdot 10^{-5} \qquad Noff := \frac{Toff}{dt} = 1 \times 10^{4}$$

$$N := 20 \cdot n - 1 \qquad i := 1 .. N \qquad t_i := i \cdot dt$$

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Ina := | F_(n+2) ← 0
       | B_(n+1) ← 0
       | for i ∈ 1 .. N − 1
       |   | Vgen ← Vg  if i > 1
       |   | if i > Noff
       |   |   | Vgen ← 0
       |   |   | Rn ← 200
       |   | (Inr  Ina) ← near(Ini, Vgen)
       |   | (F  Ifi) ← forward(F, Inr)
       |   | (Ifr  Ifa  Qfa) ← far(Ifi, Qfa)
       |   | (B  Ini) ← back(B, Ifr)
       |   | Ina_i ← Ina
       | Ina
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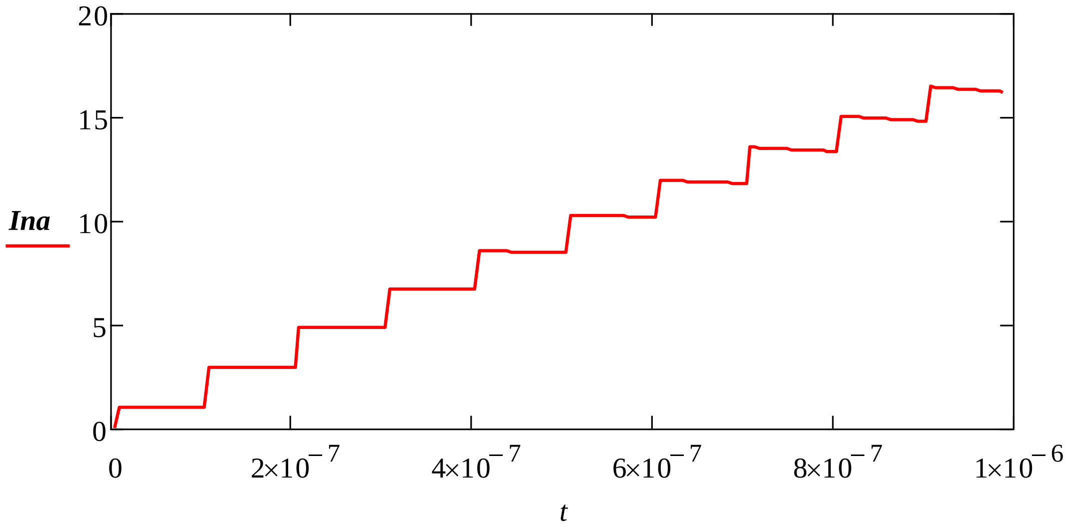


$$T = 5 \times 10^{-8} \qquad Vg = 100 \qquad Ro = 100 \qquad Rn = 1 \qquad Rf = 0 \qquad C = 5 \times 10^{-7}$$

Figure 6 Second page of Mathcad worksheet

The waveform of the input current *Ina* over a period of one microsecond is given by the graph at the bottom of Figure 6. This shows that the current delivered to the line is constant at one Amp for 100 nanoseconds; that is, for the time it takes for the step to propagate to the far end and then back to the near end.

Then the size of the step changes to 2A. This is because the capacitor is acting as a short circuit and sending the current back to the near end where it acts as a voltage source in series with the generator. Thereafter the waveform looks like a series of steps. Since the capacitor is gradually charging up, it starts to act as a voltage source at the far end. This causes a downward slope on each step.

Eventually, the amplitude of the downward slope is greater than that of the upward step, and the current reaches peak amplitude, as shown by Figure 7.

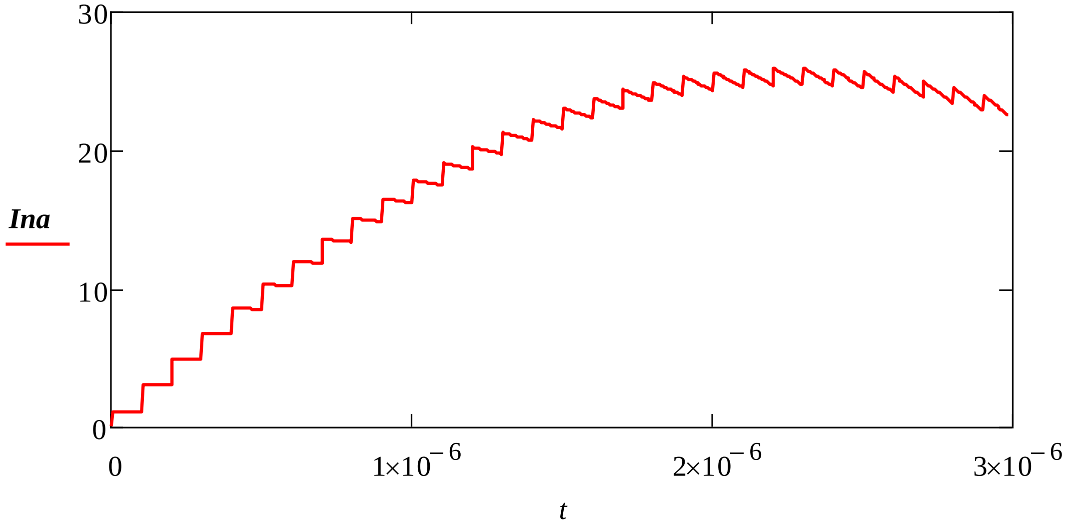


Figure 7 Waveform of current at the near end over a period of 3 micro-seconds

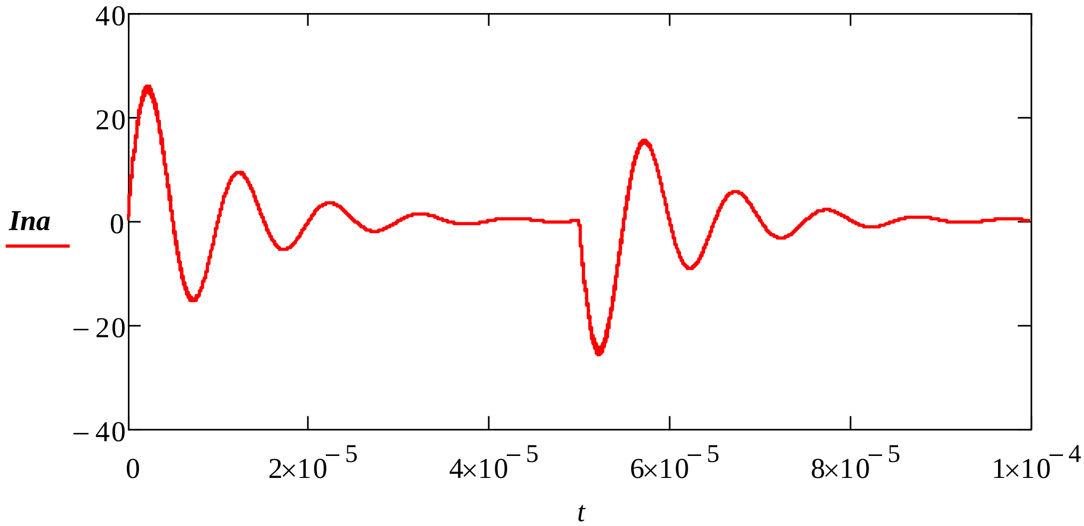


Figure 8 Waveform of current at the near end over a period of 100 micro-seconds

The result is a ringing waveform which rapidly dies down as the voltage on the capacitor settles at 100 V. Since SW1 switches to the open-circuit condition at 50 microseconds, the energy stored in the capacitor flows back into the cable and out into the environment, as illustrated by Figure 8.

In this model, it is assumed that the radiation resistance of the cable is 100 ohm. So the voltage appearing at the near end of the cable would be as shown by Figure 9.

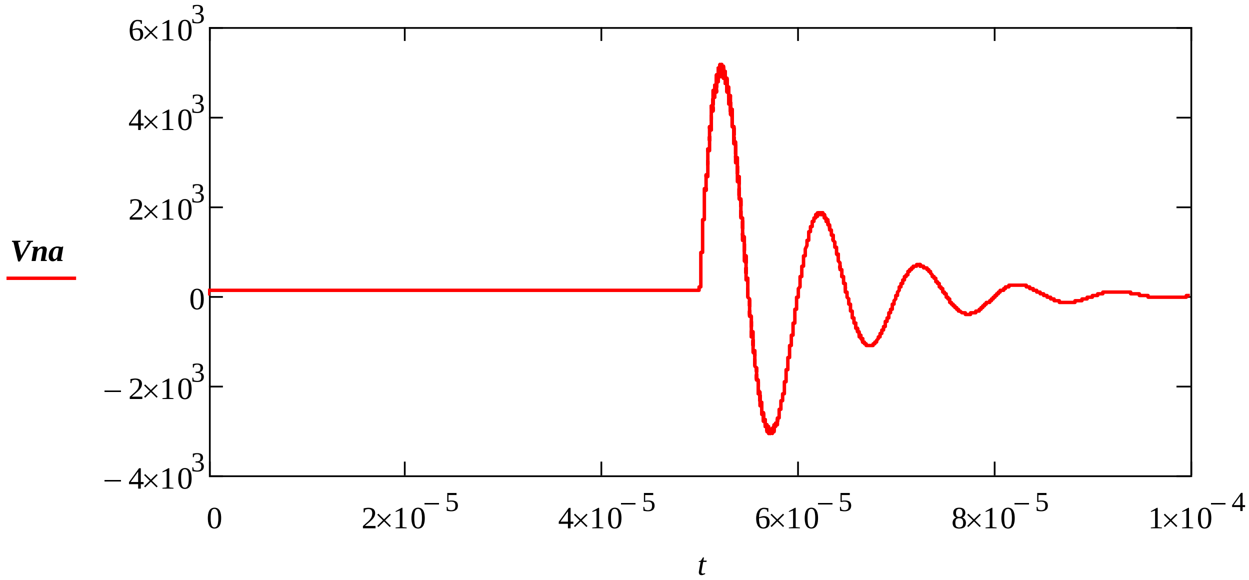


Figure 9 Voltage appearing at the near end of the cable when power is switched off

Inductive Load

Figure 1 shows the power supply line, this time with an inductive termination. When the switch closes, a current step will propagate along the cable assembly, just as with a capacitive load. But when it reaches the far end, the inductance acts as an open circuit and the voltage doubles. This voltage propagates back to the near end, which doubles the current.

To simulate the reaction of the system, it is necessary to redefine the far end equations. Figure 11 shows the circuit model.

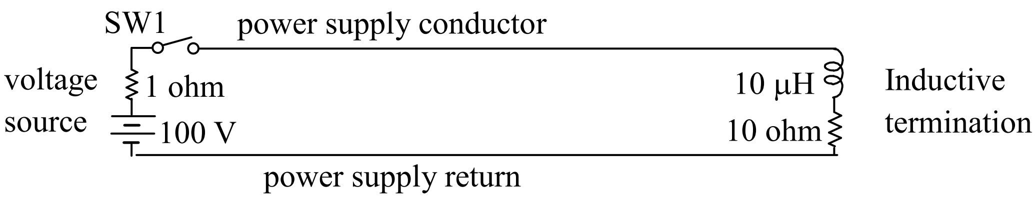


Figure 10 Power line wiring diagram, inductive termination

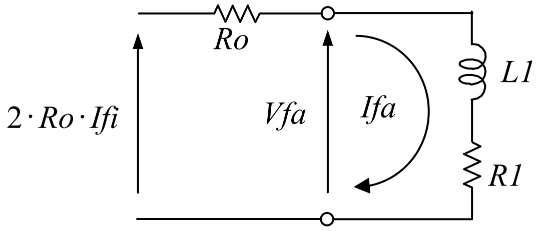


Figure 11 Circuit model which defines the response at the far end.

The mesh equation derived from Figure 11 is

$$2 \cdot Ro \cdot Ifi = (Ro + R1) \cdot Ifa + L1 \cdot \frac{dIfa}{dt} \qquad (6)$$

where *dIfa* is the amount by which *Ifa* changes during the time *dt* . This enables the function defining the response to be formulated. Figure 12 is a copy.

$$far(Ifi, Ifa) := \left| \begin{array}{l} dIfa \leftarrow \frac{dt}{L} \cdot [2 \cdot Ro \cdot Ifi - (Ro + Rf) \cdot Ifa] \\ Ifa \leftarrow Ifa + dIfa \\ Ifr \leftarrow Ifa - Ifi \\ (Ifr \quad Ifa) \end{array} \right.$$

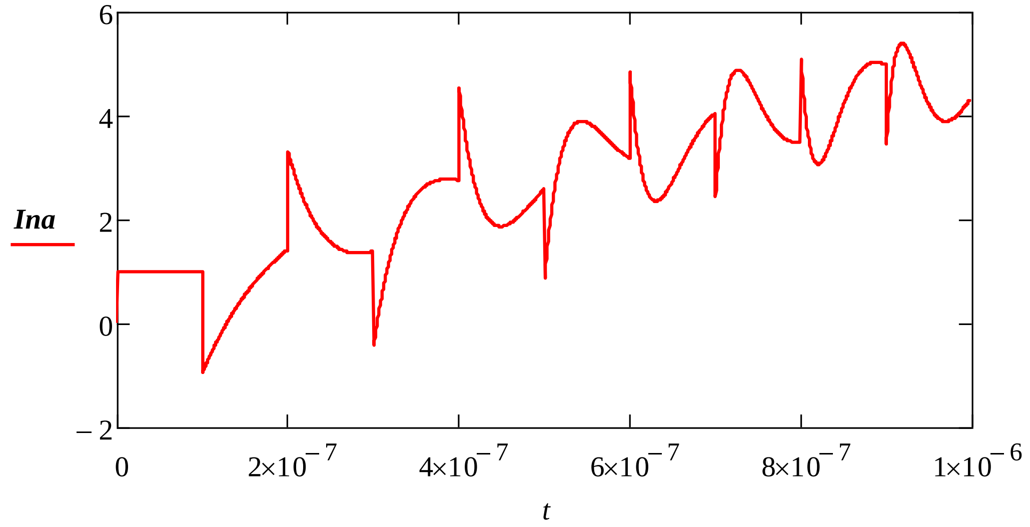
Figure 12 Mathcad function which defines the response at the far end

Modifying the Mathcad worksheet to incorporate the changes necessary to simulate the response of the system illustrated by Figure 10 and running the program leads to the simulation of Figure 13.

The waveform of the input current over a period of one microsecond is given by the graph at the bottom of Figure 13. This shows that the current delivered to the line is constant at one Amp for 100 nanoseconds; that is, for the time it takes for the step to propagate to the far end and then back to the near end.

The inductance at the far end acts as an open circuit. When the step arrives at the far end, the direction of the current reverses and the step change propagates back to the near end. In the meantime, the constant voltage applied to the inductor causes the current to rise exponentially. So the amplitude of the reflected current drops exponentially. When the pulse arrives back at the near end, and is again reflected. The waveform during the first 200 nanoseconds, and the next 800 ns, is as displayed on Figure 13.

The data listed under the graph of Figure 13 defines the parameters for the circuit of Figure 10. It was necessary to increase the number of segments *n* to 100 because the current was changing more rapidly than with a capacitive termination. Extending the period of the simulation leads to the waveform if Figure 14. The curve rises exponentially, flattening out at about 9A. In this simulation, the switch opens at 10 microseconds, the surplus energy radiates out into the environment as a burst of high frequency radiation, and the current in the load drops exponentially towards zero.



$Vg = 100 \quad Ro = 100 \quad Rn = 1 \quad Rf = 10 \quad L = 1 \times 10^{-5} \quad n = 100$

Figure 13 Waveform of current at the near end. Inductive load

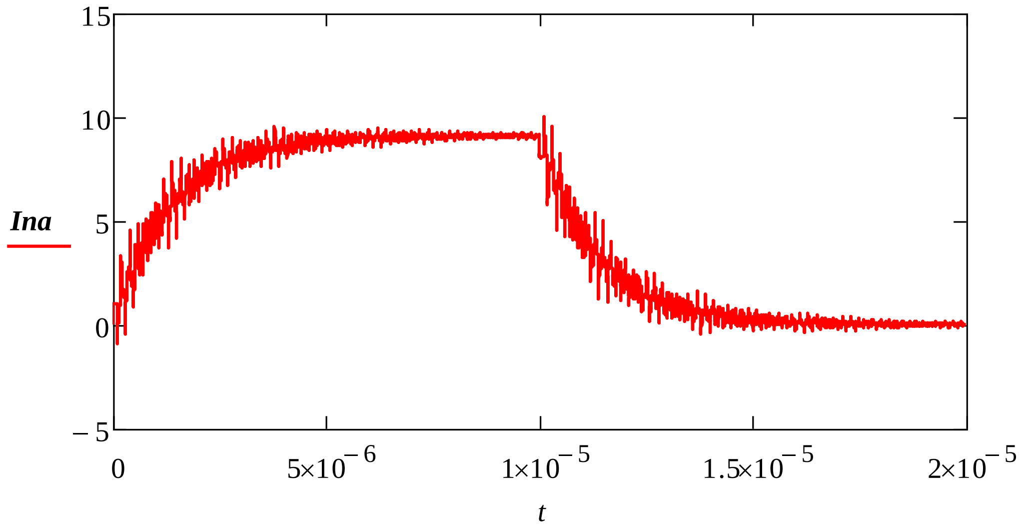


Figure 14 Waveform of the current at the near end, showing the effect of switch-off.

Figure 15 simulates the waveform of the voltage *Vna* at the near end. In radiating out into the environment, the current creates a high voltage across the terminals of the switch; of the order of one kilovolt.

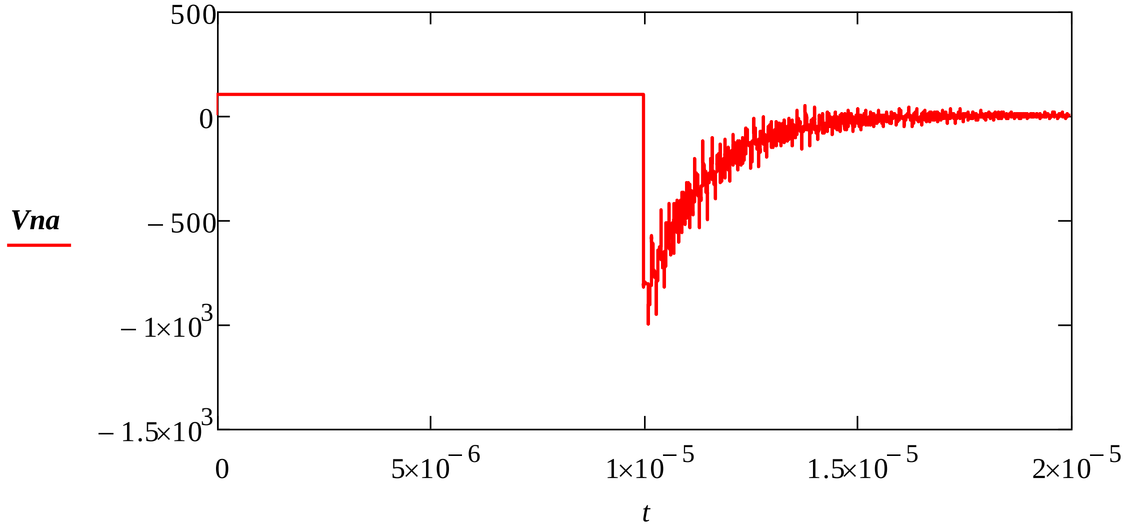


Figure 14 Waveform of voltage at the near end, showing effect of switch-off

Assessment

A capacitor at the far end of the cable acts as a short circuit to any transient current. The amplitude of the current will double. Since the source resistance at the near end is less than the characteristic impedance of the line, the current increases again and the resultant waveform is rather like a staircase. The current delivered to the capacitor builds up until it is greater than that of the generator. A ringing waveform is created. When the source is switched off, the energy stored in the system departs in the form of a high voltage ringing pulse.

An inductor at the far end acts as an open circuit. So any transient voltage will double in magnitude and the reflected voltage propagates back. Reflections at the near end create a pulsed waveform which gradually increases the current delivered to the load. Current in the cable contains a wide variety of high-frequency components. When the source is switched off, the energy departs explosively in the form of a step voltage applied along the length of the cable.

During switch-on, in both cases, current build up is in the form of a series of pulses along the line. It is inevitable that the line will act as a radiating antenna. After switch-off, in both cases, the departure of the stored energy creates a high voltage pulse at the near end.

Conclusion

A filter with reactive components will interact with the supply cable to create high frequency resonance, and this resonance will be a rich source of EMI.