Updating Circuit Theory: Charges and Photons

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Introduction

By invoking the concept of the shift register, a program is developed to simulate the propagation of charges back and forth along a cable. By adding the equations which define the incident, reflected and absorbed currents, a program is developed which can simulate the response of any transmission line to a transient waveform.

The response of an open-circuit line to a 40 ns pulse is simulated and a snapshot is provided of the distribution of current and voltage along the line after an elapsed time of 60 ns. A similar snapshot illustrates the status of a short-circuited line.

The program itself provides an insight into the mechanism involved. It reveals that voltage and current at any point at any instant are each a function of charge. Further reasoning leads to the conclusion that charge itself is a manifestation of the interaction between photons and conducting material.

Synchronisation

Figure 1 illustrates a schematic diagram of a twin-conductor transmission line. The voltage source *Vgen* at the near end of the line creates a current *In*. This current propagates along the line at a velocity similar to that of light. The current *If* arriving at the far end generates a voltage *Vf* across that load. Analysis in the frequency domain leads to the derivation of formulae for the characteristic impedance *Zo* and the phase constant λ . A significant relationship which emerges is that.

$$\frac{\delta I}{\delta x} = \frac{1}{Zo} \cdot \frac{\delta V}{dx} \tag{1}$$

Where *V* is the voltage between the send and return conductors at the point *x* and *I* is the loop current at the same location. This relationship is derived in any book on Electromagnetic Theory; or in the article 'Updating Circuit Theory – Transmission Line Model'.

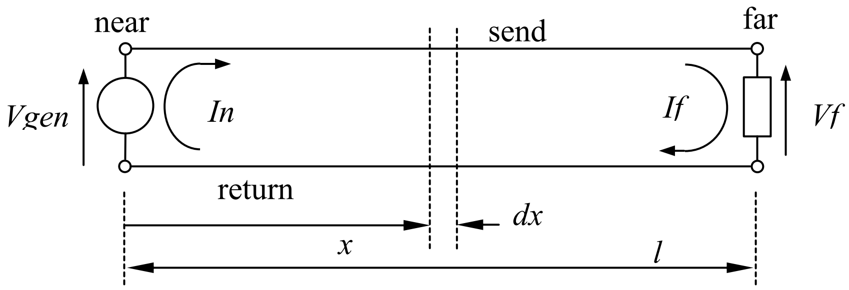

Figure 1 Schematic diagram of a twin conductor transmission line

The more familiar relationship is

$$I = \frac{1}{Zo} \cdot V \tag{2}$$

Equation (1) means that the rate of change of current is proportional to the rate of change of voltage. Equation (2) means that the current is also proportional to the voltage. The only way for these two equations to be valid is for the phase difference between current and voltage to be zero; at least during forward propagation of the signal. Figure 2 illustrates this.

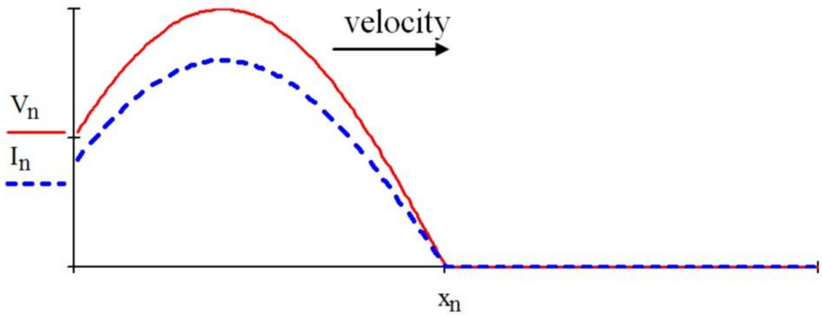

Figure 2 Relationship between voltage and current waveforms

Delay line model

Time-step analysis is based on the assumption that the current is constant for a finite time *dt*. If it is assumed that there are no losses in the line, then the assembly of Figure 1 can be modelled as a set of *n* segments, each of length

$$dx = \frac{l}{n} \tag{3}$$

If *T* is the time taken for a pulse to travel from one end of the line to the other, then the time taken for it to propagate along one segment is

$$dt = \frac{T}{n} \tag{4}$$

If a voltage *Vn* is applied to the near end of the line, then the current *In* flowing into the line would be

$$In = \frac{Vn}{Ro} \tag{5}$$

Where

$$Ro = \sqrt{\frac{L}{C}} \tag{6}$$

and where L is the inductance of the conductor pair and C is the capacitance between those conductors. During the time dt the charge Qni delivered to one segment by the incident current Ini is

$$Qni = Ini \cdot dt \tag{7}$$

This charge propagates along the line at a velocity v

$$v = \frac{dx}{dt} = \frac{l}{T} \tag{8}$$

At the far end, some of this charge is delivered to the interface circuitry at the receiver. What is not absorbed is reflected. The reflected charge Qf travels back to the near end and appears as an incident voltage Vni. In terms of electromagnetic theory, this energy is carried by an electromagnetic wave, as illustrated by the block diagram of Figure 3

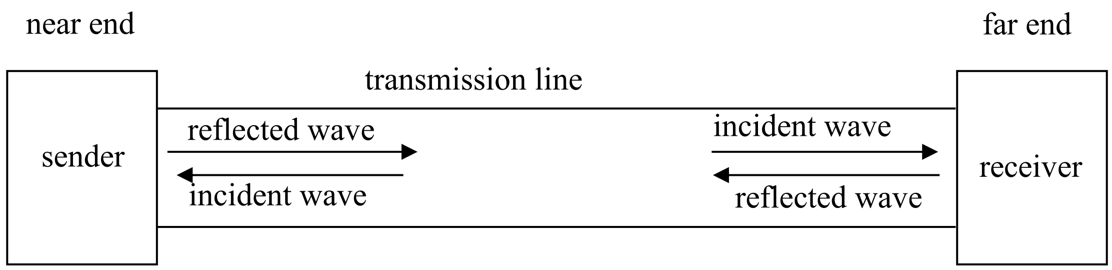

Figure 3 Block diagram illustrating the propagation of electromagnetic energy.

Reflections.

There are now two voltage sources appearing at the near end: The generator $Vgen$ and the voltage Vni delivered by the incident wave. To determine the response at this end, it is necessary to define the parameters and their relationships.

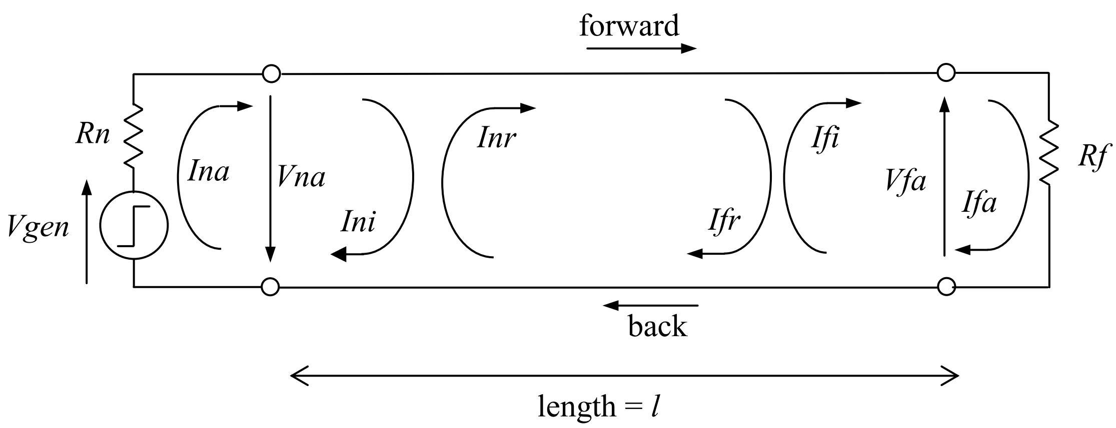

Figure 4 Delay line model

In Figure 4, *Ini* is the incident current arriving at the near end and *Inr* is the current reflected back into the line. The current absorbed at the near end is *Ina*. Similarly, *Ifi*, *Ifr* and *Ifa* are incident, reflected and absorbed currents at the far end. These currents are depicted as loop currents flowing along the send conductor and back via the return conductor. All current loops are defined as clockwise. Rn is the source resistance at the near end and Rf is the load resistance at the far end. The relationships between currents and voltages are:

$$Vni + Vnr = Vna \tag{9}$$

$$Inr + Ini = Ina \tag{10}$$

$$Vni = Ro \cdot Ini \tag{11}$$

$$Vnr = -Ro \cdot Inr \tag{12}$$

$$Vna + Vgen = Rn \cdot Ina \tag{13}$$

Adding (11) and (12) and invoking (10)

$$Vni - Vnr = Ro \cdot Ina \tag{14}$$

Using(13) to substitute for *Vna* in (9)

$$Vni + Vnr = Rn \cdot Ina - Vgen \tag{15}$$

Adding (14) and (15)

$$2 \cdot Vni = (Ro + Rn) \cdot Ina - Vgen \tag{16}$$

Using (11) to substitute for *Vni* in (16) and rearranging

$$Ina = \frac{2 \cdot Ro \cdot Ini + Vgen}{Ro + Rn} \tag{17}$$

This means that, if the value of *Ini* is known, then the value of *Ina* can be calculated. Invoking (10) allows the value of *Inr* to be determined. The charge *Qnr* delivered to the first segment of the line can be calculated using equation (7). This gives the value of the charge *Qfi* arriving at the far end after the elapsed time *T*. (At any instant, the values of *Qnr* and *Qfi* are usually quite different)

The current *Ifi* arriving at the far end is:

$$Ifi = \frac{Qfi}{dt} \tag{18}$$

The current at the far end can be calculated using the relationship

$$Ifa = \frac{2 \cdot Ro \cdot Ifi}{Ro + Rf} \tag{19}$$

Computaion

Having defined all the relationships involved in simulating the response of the line, the next step is to create a Mathcad worksheet to simulate the mechanism involved.

Figure 5 is a copy of the first page. As with any program, the first action is to define the variables used on the page; the voltage *Vg* of the generator, the characteristic resistance *Ro*, and the values of the resistors *Rn* and *Rf* at the near and far ends. It is assumed that the length of the line *l* is 15 m and that the velocity of propagation *v* is 300 m/micro-sec. This gives a value for the time *T* it takes for a pulse to travel from one end to the other. It is assumed that the number of segments *n* of the line is 100. This gives a value of *dt*, the time it takes a charge to move from one segment to the next.

The function *near*(*Qni* , *Vg*) invokes equations (7) and (17) to calculate the value of *Ina*, then equations (10) and (7) to evaluate *Qnr*. The output of this function is a vector containing the variables *Qnr* and *Ina*.

The parameter ***F*** in the input to the function *forward*(***F*** , *Qn*) is a vector of *n* lines and one column. It holds the value of the charge in every segment of the line. The first action of the function is to place the value *Qn* in the first segment. Then it moves the value of the charge in every segment to the next segment. This leaves the first segment free to accept the next input. The value of the parameter *Qf* in the final segment is provided as one output of the function. The other output is the new set of values in the vector ***F***.

The function *far*(*Qfi*) takes as input the value of the charge delivered to the end of the line. Then it calculates the value of the current *Ifa* and that of the charge *Qfr* reflected back into the line. These values are provided as the output of this subroutine.

The function *back*(***B***, *Qf*) processes the value of *Qf* in much the same was as *forward*(***F*** , *Qn*) shifts *Qn* forward one segment. But this time, the process is reversed. *Qfr* is shifted back one segment. This means that the output *Qni* arriving at the near end of the line is that which was delivered to the far end at time $\left(t-T\right)$.

The functions *forward*(***F*** , *Qn*) and *back*(***B***, *Qf*) behave in essentially the same was as shift registers.

The main program is defined on Figure 6, the second page of the Mathcad worksheet. This simulates the behaviour of the transmission line when a single pulse of 100 V is applied to the line via a 10 ohm resistor (*Rn*). The duration of this pulse, *Tmark* is defined as 40 ns. Since the time taken for a charge to move from segment to segment is 0.5 ns, the number of segments, *width*, to which this charge is delivered, is 80.

$Vg := 100 \quad Ro := 100 \quad Rn := 10 \quad Rf := 10^7$

$l := 15 \quad v := 3 \cdot 10^8 \quad T := \frac{l}{v} = 5 \times 10^{-8}$

$n := 100 \quad dt := \frac{T}{n} = 5 \times 10^{-10}$

$$near(Qni, Vg) := \left| \begin{array}{l} Ini \leftarrow \frac{Qni}{dt} \\ Ina \leftarrow \frac{2 \cdot Ro \cdot Ini + Vg}{Ro + Rn} \\ Qnr \leftarrow (Ina - Ini) \cdot dt \\ (Qnr \quad Ina) \end{array} \right.$$

$$forward(\boldsymbol{F}, Qn) := \left| \begin{array}{l} \boldsymbol{F}_1 \leftarrow Qn \\ for \;\; x \in n+1 .. 1 \\ \quad \boldsymbol{F}_{x+1} \leftarrow \boldsymbol{F}_x \\ Qf \leftarrow \boldsymbol{F}_{n+2} \\ (\boldsymbol{F} \quad Qf) \end{array} \right.$$

$$far(Qfi) := \left| \begin{array}{l} Ifi \leftarrow \frac{Qfi}{dt} \\ Ifa \leftarrow \frac{2 \cdot Ro \cdot Ifi}{Ro + Rf} \\ Qfr \leftarrow (Ifa - Ifi) \cdot dt \\ (Qfr \quad Ifa) \end{array} \right.$$

$$back(\boldsymbol{B}, Qf) := \left| \begin{array}{l} \boldsymbol{B}_{n+1} \leftarrow Qf \\ for \;\; x \in 2 .. n+1 \\ \quad \boldsymbol{B}_{x-1} \leftarrow \boldsymbol{B}_x \\ Qn \leftarrow \boldsymbol{B}_1 \\ (\boldsymbol{B} \quad Qn) \end{array} \right.$$

Figure 5 First page of Mathcad worksheet

$$Tmark := 40\cdot 10^{-9} \qquad width := \text{floor}\left(\frac{Tmark}{dt}\right) = 80$$

$$Tfreeze := 60\cdot 10^{-9} \qquad Nfreeze := floor\left(\frac{Tfreeze}{dt}\right) = 120$$

$$N := 2\cdot n - 1 \qquad i := 1 .. N \qquad t_i := i\cdot dt \qquad x_i := i\cdot\frac{l}{n}$$

Main Programme

$$(\boldsymbol{Qfd} \quad \boldsymbol{Qbk}) := \left|\begin{array}{l} \boldsymbol{F}_{n+2} \leftarrow 0 \\ \boldsymbol{B}_{n+1} \leftarrow 0 \\ for \ \ i \in 1 .. N - 1 \\ \quad \left|\begin{array}{l} Vgen \leftarrow Vg \ \ if \ \ i > 1 \\ Vgen \leftarrow 0 \ \ if \ \ i > width \\ (Qnr \quad Ina) \leftarrow near(Qni, Vgen) \\ (\boldsymbol{F} \quad Qfi) \leftarrow forward(\boldsymbol{F}, Qnr) \\ (Qfr \quad Ifa) \leftarrow far(Qfi) \\ (\boldsymbol{B} \quad Qni) \leftarrow back(\boldsymbol{B}, Qfr) \\ if \ \ i = Nfreeze \\ \quad \left|\begin{array}{l} \boldsymbol{Qfd} \leftarrow \boldsymbol{F} \\ \boldsymbol{Qbk} \leftarrow \boldsymbol{B} \end{array}\right. \end{array}\right. \\ (\boldsymbol{Qfd} \quad \boldsymbol{Qbk}) \end{array}\right.$$

$$(\boldsymbol{Ix} \quad \boldsymbol{Vx}) := \left|\begin{array}{l} for \ \ x \in 1 .. n + 1 \\ \quad \left|\begin{array}{l} \boldsymbol{I}_x \leftarrow \dfrac{(\boldsymbol{Qfd}_x + \boldsymbol{Qbk}_x)}{dt} \\ \boldsymbol{V}_x \leftarrow \dfrac{(\boldsymbol{Qfd}_x - \boldsymbol{Qbk}_x)}{dt}\cdot Ro \end{array}\right. \\ (\boldsymbol{I} \quad \boldsymbol{V}) \end{array}\right.$$

Figure 6 Second page of Mathcad worksheet

The simulation shows that the leading edge of the pulse propagates forward to the far end, is reflected, and then propagates back towards the near end. The objective of this particular simulation is to determine the distribution of charges along the line at the instant $Tfreeze = 60$ ns. This is essentially the same as freezing a video picture. Hence the number of time steps which will have occurred up to this instant would be *Nfreeze*. This is equivalent to 120 line segments. (The Mathcad function ‘floor’ ensures that the value returned is a whole number).

The number *N* of time steps of the simulation ensures that pulse at $t = 1$ will arrive back at the near end. The time taken is $2 \cdot T$. The vector *t* provides a value for the horizontal axis of any graph of the waveform of any voltage or current. The vector *x* allows the distribution of current or voltage along the line to be depicted.

The program first defines the vector ***F*** which holds the value of the forward moving charge at every segment of the line at any instant. The vector ***B*** holds the values of the charges moving back along the line.

The repetitive part of the program first defines the waveform of the generator voltage, *Vgen*. Then it calculates the values of the current at each termination and the charges located at every segment at every instant. The output is two vectors; ***Qfd*** holds the values of the forward-moving charge at each line segment at the time *Tfreeze*, and ***Qbk*** holds the values of the charges moving back towards the near end.

The final function defined on Figure 5 calculates the sum and difference between the forward and backward moving charges and computes the value of the current ***Ix*** and the voltage ***Vx*** at each location of the line at the instant *Tfreeze*.

Since the charge Qfd_x is propagating forward whilst Qbk_x is flowing backward, then the sum of these parameters gives a value for the amount of charge flowing along that particular segment. (*Qbk* is a negative quantity since it is flowing backwards). Equally, the difference between these parameters gives a value for the charge accumulated at that location.

Results

Figure 7 depicts the graphs of the variation of *Vx* and *Ix* with distance *x* along the length of the line. *Vx* is the voltage between the conductors and *Ix* is the value of the current flowing forward along the send conductor and back via the return conductor. The leading edge is propagating back towards the near end of the line, whilst the trailing edge s still moving forward. The voltage at the far end doubles, whist the current drops to zero. The arrows illustrate the direction of propagation of each edge of the waveform

Figure 8 depicts these same two parameters when the resistance at the far end of the line is zero. That is, when the terminals at the far end are shorted together. This time, the voltage at the far end drops to zero whilst the amplitude of the current doubles.

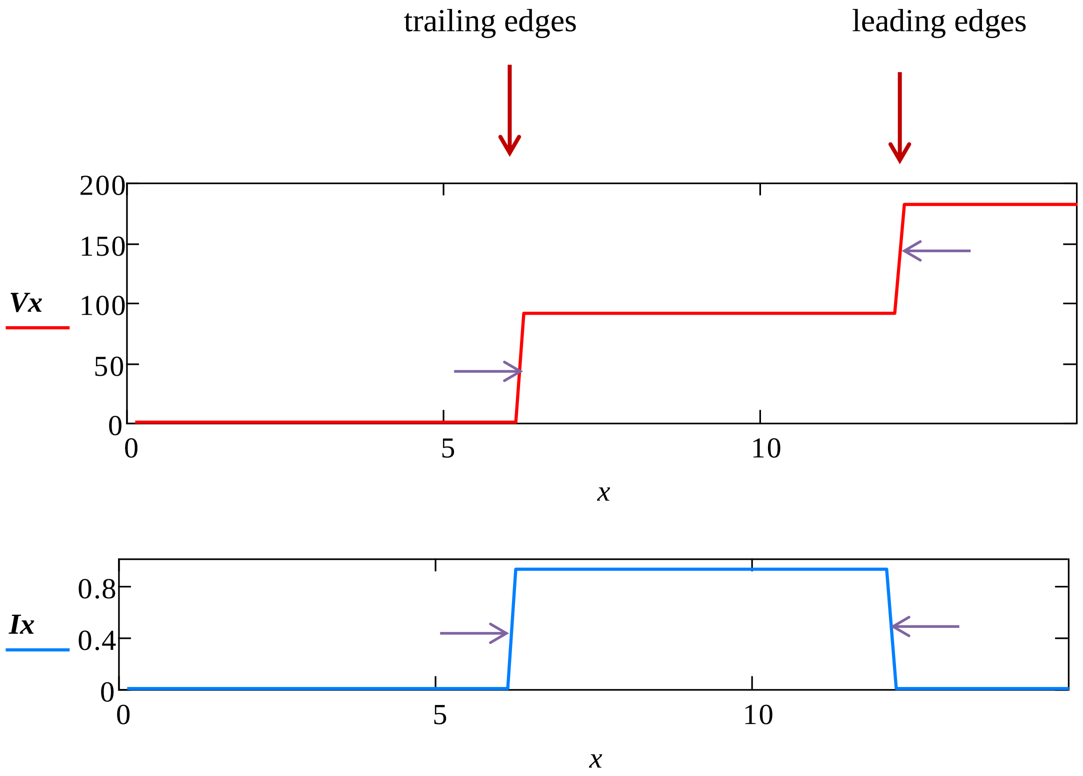

Figure 7 Distribution of voltage and current at *t* = 60ns. Far end open-circuit.

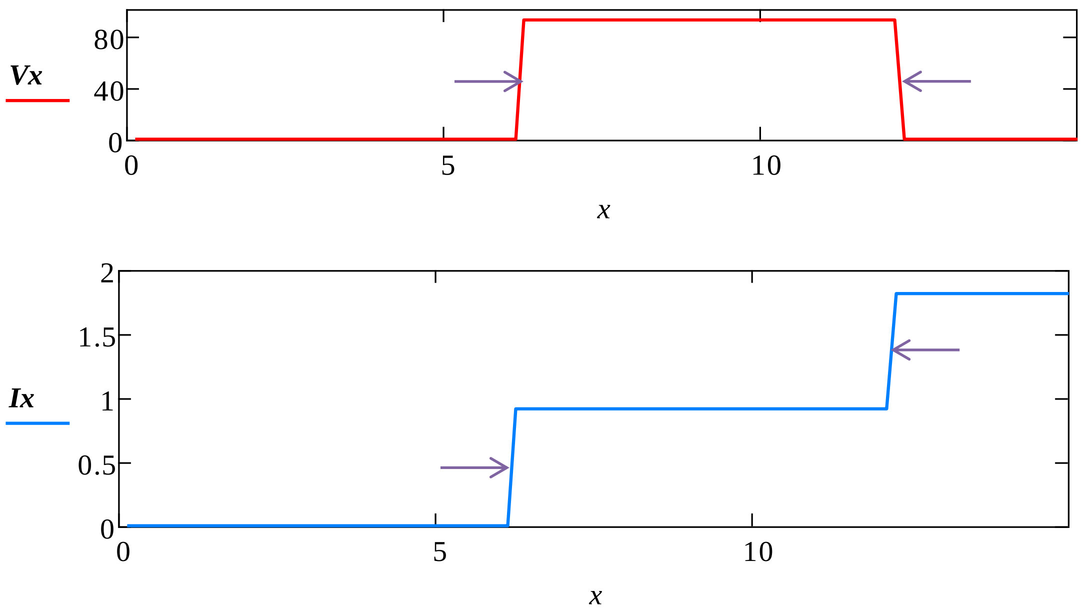

Figure 7 Distribution of voltage and current at *t* = 60ns. Far end short-circuit

Assessment

The program defined in Figures 5 and 6 calculates the amplitude of the current *Ina* at the near end of the line. This current flows in the resistor *Rn*. It is possible to calculate the value of the voltage across this resistor at any instant. The same is true of the current and voltage at the far end. Any of these parameters can be selected as the output of the main program. The waveform of the input voltage *Vgen* can be defined as a voltage which varies with time. So it is possible to simulate the response of the signal link between the sender and receiver to any transient waveform

Underlying mechanism

The results illustrated on Figures 7 and 8 are entirely in accordance with the conclusions of transmission line theory. Voltage doubles at an open circuit; current doubles at a short circuit.

The significance of this illustration is that it identifies the fact that voltage and current are manifestations of the action of charges. Voltage is a function of the number of charges accumulated at a particular segment. Current is a function of the flow of charges through that segment.

But charged particles do not move at the velocity of light along conductors. Nor do they move freely through the insulating material enclosing conductors. The entities with that ability are photons, and photons do not carry charge. But they can apply force. The logical conclusion must be that charge itself is a function of the interaction of photons with atoms. When they interact, a force is applied to the conducting material and hence, between the conductors.

However, it is unnecessary to understand the intricacies of quantum physics to design electronic circuitry. Nor is it necessary to be able to handle the complexities of full-field analysis, as developed by electromagnetic theory.

The concept of the shift register is well understood by circuit designers. Invoking this concept allows the mechanisms involved in the transmission of energy along and between transmission lines to be analysed using the equations of circuit theory.

Conclusion

A program has been developed which allows the transient response of any transmission line to be simulated.

The analysis also identifies the fact that currents and voltages are manifestations of the action of charges. Charges themselves are due to the interaction of photons with conductors.