Updating Circuit Theory – Modelling Radiated Interference

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Introduction

A circuit model is derived which can simulate the differential-mode response of a signal link when it is exposed to electromagnetic interference.

The concept of the radiation resistance is developed to relate the current in an isolated conductor to that of the current in a dipole transmitting antenna. By invoking worst-case conditions, a simple equation can be used to define the maximum H-field of the transmitted wave in terms of the current in the conductor. The E-field can then be determined by multiplying the H-field by the intrinsic impedance of free space. When the E-field interacts with a conductor in the far distance, a voltage is induced in that conductor. A simple equation is derived which relates that voltage to the magnitude of the E-field. The power received is then calculated by dividing the square of the voltage by the radiation resistor.

Representing a twin-conductor cable as two transmission lines in series allows a circuit model to be created which simulates the differential-mode current induced in the signal link.

Emission

Figure 1 illustrates the model of an isolated conductor when a voltage source *Vgen* is located half-way along the length. This model treats the conductor as two monopoles. At low frequencies, the current *I* will be negligible; but as the frequency increases towards resonance, this current will increase rapidly. At resonance, the current is limited only by the resistance *Rrad*. The article 'Updating Circuit Theory – The Radiation Resistance' provides an example of the response of a 15 metre conductor. The parameters *Lp* and *Cp* represent the properties of a single monopole of length 7.5 metres and the parameter *Rrad* represents the effect of current radiating out into the environment in the form of an electromagnetic field.

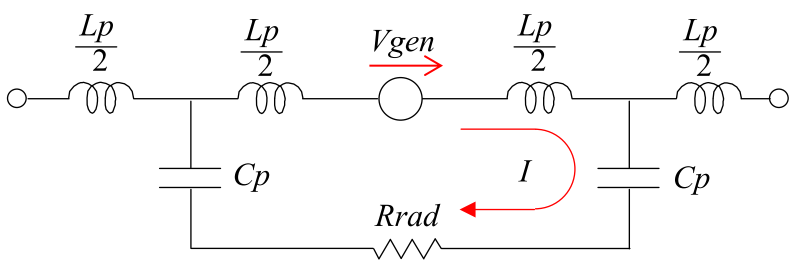

Figure 1 Simulating radiated emission

Propagation

Figure 2 shows the relationship between the electric field E and magnetic field H at a point on the surface of a sphere of radius r. The power density S is the product:

$$S = E \cdot H \quad \text{Watts per square metre} \tag{1}$$

This diagram also shows that these three vectors are perpendicular to each other and that the energy propagates outwards along the radius r. These are the only components of the electromagnetic field which radiate out into the far distance.

The power density varies with latitude θ and is a maximum when $\theta = 90$ degrees. With any analysis of electromagnetic interference it is maximum values which are most critical. EMC tests are usually defined in terms of pass/fail criteria. If the measured parameter is less than a specified limit, then the system under review will pass that particular test. Hence it is only necessary to analyse worst-case conditions.

In this case, it can be assumed that the maximum magnetic field H is in the x-y plane and the maximum electric field E is parallel to the z axis. So

$$H = \frac{I}{2 \cdot \pi \cdot r} \quad \text{ampere per metre} \tag{2}$$

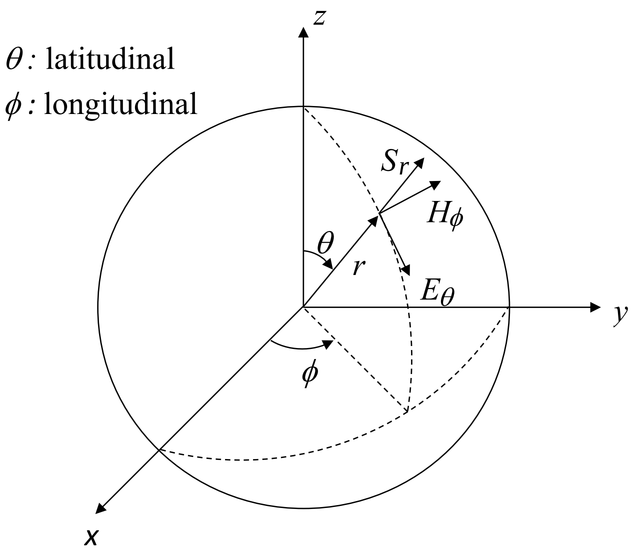

Figure 2 Power density on a spherical surface.

The characteristic impedance of free space is

$$Zo = \frac{E}{H} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 377 \quad \text{ohm} \tag{3}$$

Hence the maximum value of the electric field is

$$E = Zo \cdot H \quad \text{Volt per metre} \tag{4}$$

Substituting for *H*

$$E = Zo \cdot \frac{I}{2 \cdot \pi \cdot r} \quad \text{Volt per metre} \tag{5}$$

If the amplitude of the current at the source and the separation distance from that source are known, then the maximum intensity of the electric field can be calculated using equation (5).

The Threat Voltage

If it is assumed that the same conductor now behaves in essentially the same way as a dipole receiver when it is exposed to an external electromagnetic field *E*, then this effect can be represented as the sum of a number of discrete voltage sources. An incremental voltage *dV* is induced in series with each element *dz* of the conductor. Figure 3 illustrates this.

The incremental voltage *dV* is

$$dV = E \cdot dz \tag{6}$$

If it is assumed that the waveform is sinusoidal, then the distribution in space will also be sinusoidal. Figure 4 illustrates this. For such a waveform, the relationship can be defined as

$$E = Emax \cdot \cos(2 \cdot \pi \cdot \frac{z}{\lambda}) \tag{7}$$

Where λ is the wavelength. Over the length of the cable the total voltage is:

$$Vthreat = \int_{-l}^{l} Emax \cdot \cos(2 \cdot \pi \cdot \frac{z}{\lambda}) \cdot dz \tag{8}$$

That is

$$Vthreat = Emax \cdot \frac{\lambda}{\pi} \cdot \sin(2 \cdot \pi \cdot \frac{l}{\lambda}) \tag{9}$$

If $l > \frac{\lambda}{4}$ then the waveform will include negative values, as illustrated by Figure 4. The curve relating *Vthreat* to wavelength will have a series of peaks and troughs. The envelope curve which touches the peaks can be obtained by setting

$$\sin(2 \cdot \pi \cdot \frac{l}{\lambda}) = 1 \tag{10}$$

then

$$Vthreat = Emax \cdot \frac{\lambda}{\pi} \tag{11}$$

Equation (11) defines worst-case conditions.

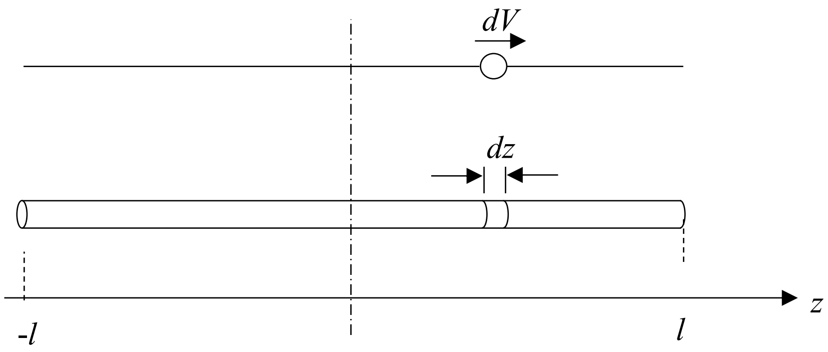

Figure 3 Conductor represented as two dipoles

Figure 3 Effect of an external electromagnetic field on a conductor EMC requirements tend to pecify the electric field strength and a range of frequencies to which the equipment under test (EUT) must be subjected and require that the EUT operates normally under these conditions. The threat voltage can be calculated using equation (11) and by invoking the relationship:

$$\lambda = \frac{c}{f} \tag{12}$$

where *c* is the velocity of light in a vacuum

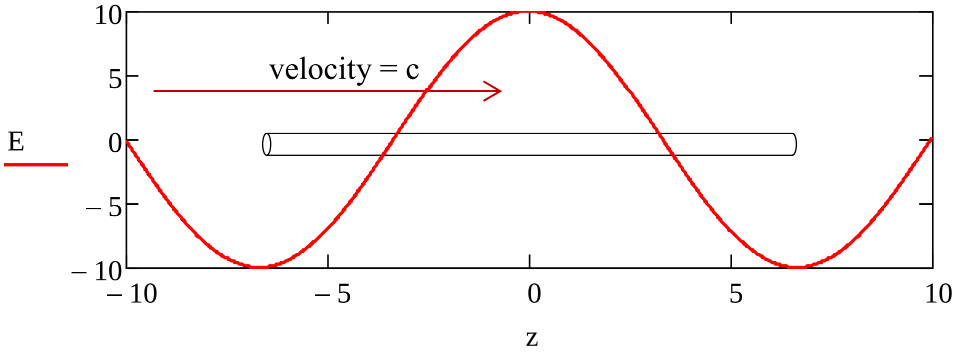

Figure 4 Distribution of electric field along conductor

Power Received

With a dipole receiving antenna as illustrated by Figure 5, the power delivered to the radio frequency receiver can be modelled as shown on Figure 6.

For this configuration, the power received *Prec* at the amplifier input when the dipole is operating at its half-wave frequency is

$$Prec = \frac{1}{Rrec} \cdot \left(\frac{Vthreat}{2} \right)^2 \tag{13}$$

If the wiring of the system-under-review is not configured as a dipole receiving antenna, then the power received will be:

$$Prec = \frac{Vthreat^2}{Rrec} \tag{14}$$

This is because the energy is stored in the conductor in the form of oscillating current, rather than delivered to the input of the RF amplifier.

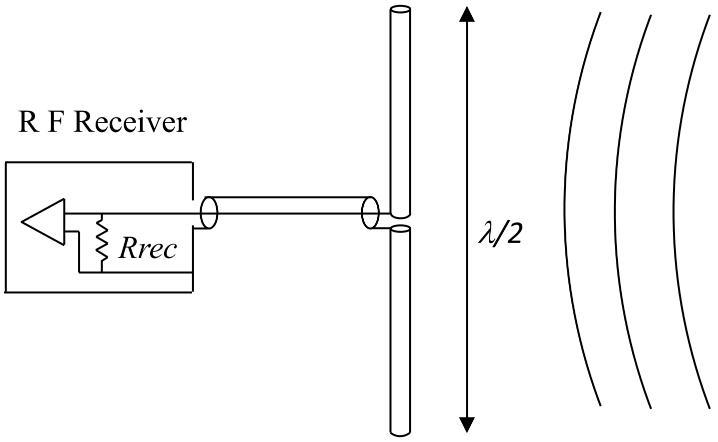

Figure 5 Illustration of a receiver assembly

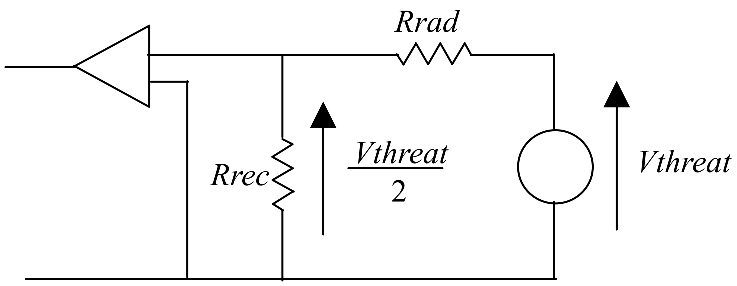

Figure 6 Circuit model of receiver assembly

The Signal Link

In any practical configuration, the cable is connected to a signal source which is mounted on a conducting framework, as illustrated by Figure 7. This shows a setup where a processing unit is transmitting a signal along a twin conductor cable to a remote transducer. It is assumed that the length *l* of the cable is 15 metre and that the amplitude *E* of the interference is 10 volts per metre. Under worst-case conditions, the length of the structure would be the same as that of the cable. So, maximum voltage would exist along the cable when the frequency of the external radiation was creating half-wave resonance along the assembly. That is, when it was creating quarter-wave resonance along the cable.

Since resonance is due to charges propagating back and forth between the extremities of the assembly, it takes several cycles of the incoming wave before the energy stored in the cable builds up and the current reaches a peak level. (This does not take very long, since the charges are moving at near-light velocity.) At frequencies lower than that of the quarter-wave resonance of the assembly, the impulses supplied by the threat field cannot sustain resonance. So the induced current is reduced.

It is fair to assume that the threat voltage is zero under steady-state conditions and increases as the frequency of the interference increases, up to that of quarter-wave resonance *fq*, and decreases as the frequency increases further.

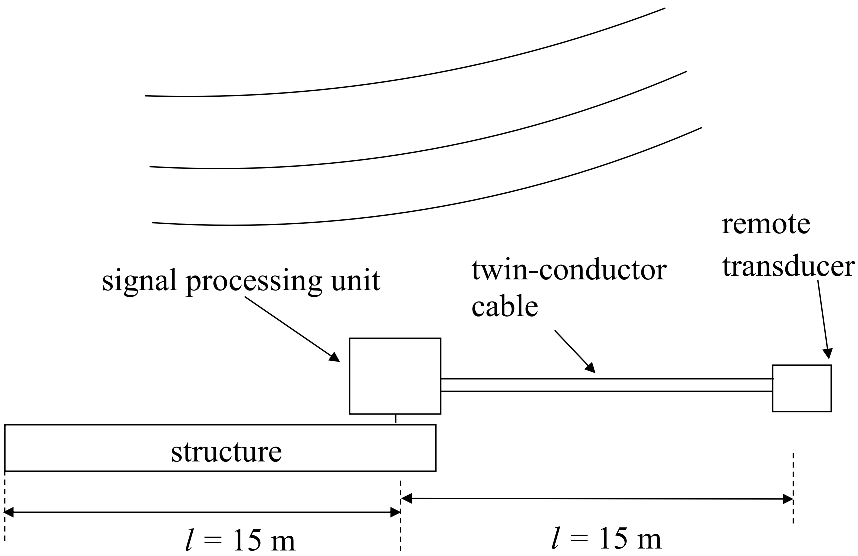

Figure 7 Cable exposed to external radiation

At quarter-wave resonance, the wavelength λq is

$$\lambda q = 4 \cdot l \qquad (15)$$

the frequency is

$$fq = \frac{c}{\lambda q} \qquad (16)$$

and the threat voltage is

$$Vq = \frac{\lambda q}{\pi} \cdot E \qquad (17)$$

at frequencies less than *fq*

$$Vthreat = Vq \cdot \frac{f}{fq} \qquad (18)$$

and at frequencies higher than *fq*

$$Vthreat = Vq \cdot \frac{fq}{f} \qquad (19)$$

The graph at the bottom of the Mathcad worksheet of Figure 8 illustrates how the amplitude of *Vthreat* varies with frequency.

$c := 300 \cdot 10^6$ $\quad E := 10$ $\quad l := 15$

$\lambda q := 4 \cdot l = 60$ $\quad fq := \frac{c}{\lambda q}$ $\quad Vq := \frac{\lambda q}{\pi} \cdot E = 190.986$

$i := 1..950$ $\quad F_i := i \cdot 10^5$

$$Vthreat_i := \left| \begin{array}{l} f \leftarrow F_i \\ V \leftarrow Vq \cdot \frac{f}{fq} \\ V \leftarrow Vq \cdot \frac{fq}{f} \quad \text{if } f > fq \end{array} \right.$$

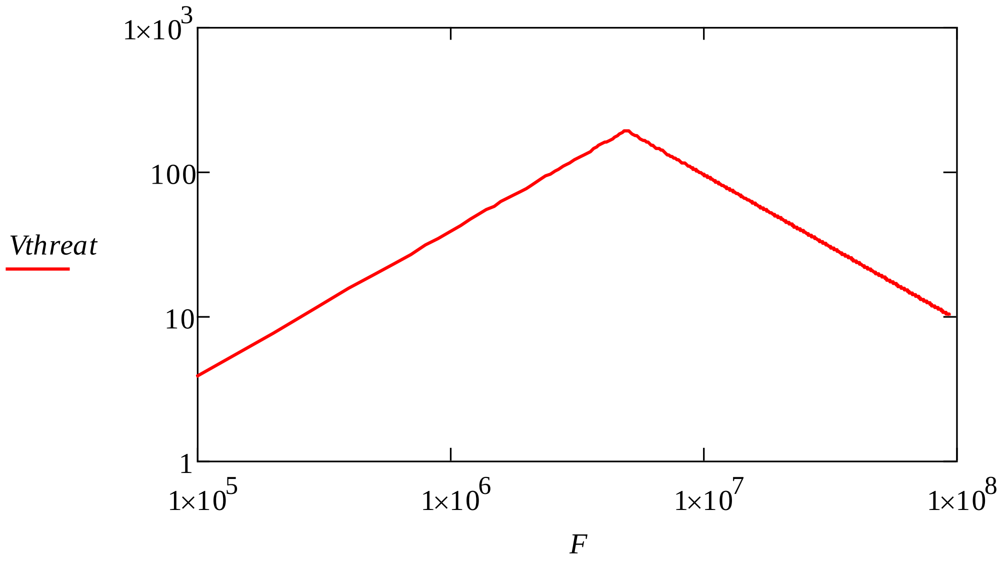

Figure 8 Graph of the variation of threat voltage with frequency

Circuit model

The model of Figure 6 indicates that the action of an external electromagnetic field on a dipole receiver can be represented as a voltage source $\frac{Vthreat}{2}$ across the input resistor. With a single conductor, the action of the field can be represented by a voltage source *Vthreat* in series with *Rrad*; both in series with the conductor.

For the two-conductor signal link depicted in Figure 7, the response of the system to the presence of an external electromagnetic field can be analysed using the circuit model of Figure 9

This is effectively a model of two transmission lines connected in series, with the voltage source located at the interface between them and connected in such a way that it creates a current along the entire length of one of the conductors. Coupling between the conductors creates a differential-mode current I_2 . In turn, this creates the currents I_1 and I_3 which flow in the resistors *Rn* and *Rf*. The relationship between the impedance parameters and the L, C, and R components is defined in the article 'Updating Circuit Theory: The Radiation Resistance.'

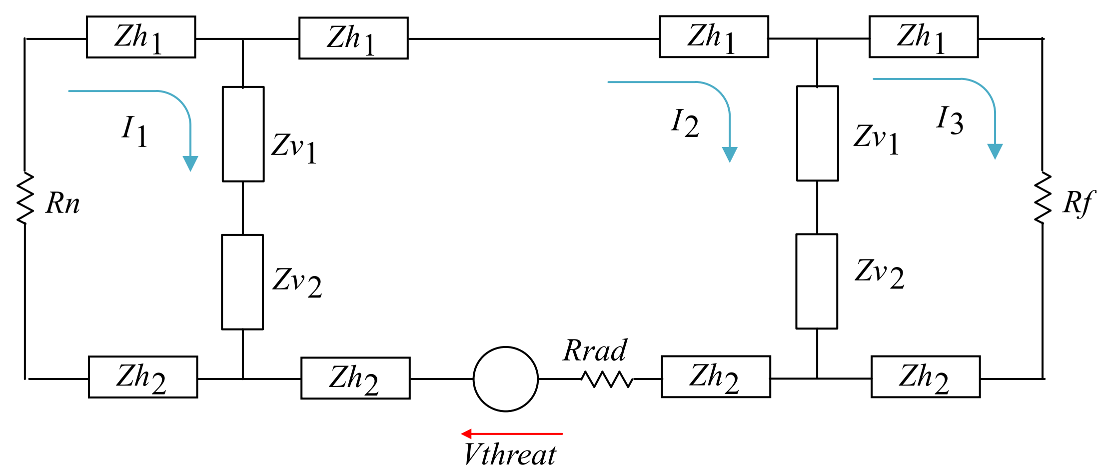

Figure 9 Simulating the response of a twin conductor line to external interference

If the amplitude of *Vthreat* and the value of the resistor *Rrad* are set at zero, then this model will behave as a transmission line. Replacing the components *Rn* and *Rf* with circuitry at the interfaces at the near and far ends will allow the functional response of this signal link to be simulated.

It then becomes possible to relate the response of the signal link under normal conditions to its response in the presence of interference.

Conclusion

A step-by-step method has been described which introduces all the key parameters involved in propagating an electromagnetic field from the source of interference to the conductors which experience that interference. The relationship between the current in the source and the magnetic field has been defined in terms of a circuit model, as has the relationship between the voltage induced in the victim circuit and the electric field.

Since worst-case analysis has been employed, the relationships are the simplest possible. It also means that the simulated response can be compared directly with the required EMC performance of the system-under-review. This opens up the way for radiated emission and radiation susceptibility to be subjected to the same design process as any other system requirement.