Updating Circuit Theory - The Virtual Conductor

Ian Darney

Introduction

A circuit model is developed to simulate the emission from a twin conductor cable. Antenna-mode current flows out into a virtual conductor.

Three of the building blocks of all circuit models are the primitive inductance, the primitive capacitance, and the series resistance. The primitive inductance relates the energy level of the magnetic field caused by current in a conductor to the voltage developed along that conductor. The primitive capacitance relates the voltage on the surface of a conductor to the electric field. The series resistor relates the current and voltage along a conductor to the power loss due to thermal effects.

The fourth building block is the radiation resistance. This simulates the power loss due to current flowing out into the environment.

This article brings all four parameters together in the form of a simple network.

The Single Conductor

The article on the Radiation Resistance started with the assumption that a length of single-core conductor with a voltage source located at the centre could be represented by the circuit model of Figure 1a. Then it described a test which enabled the values of all the components to be measured. The focus here is on the theoretical derivation for such a model.

The article 'The Three Conductor Model' derives formulae for the reactive components of Figure 1a.

$$Lp = \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{l}{r}\right) \tag{1}$$

$$Cp = \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{l}{r}\right)} \tag{2}$$

where r is the radius of the conductor, μ is the permeability of the conductor, ε is the permittivity of the insulation and l is the length of the conductor-under-review. The parameter *Rrad* represents the effective resistance of the environment. Voltage developed across *Rrad* is a measure of the energy loss due to radiation. The parameter *Rp* simulates the series resistance of each section of the conductor.

If the voltage along each section is assumed to be the same, the voltage at the centre can be defined as zero volts. Figure 1b illustrates this.

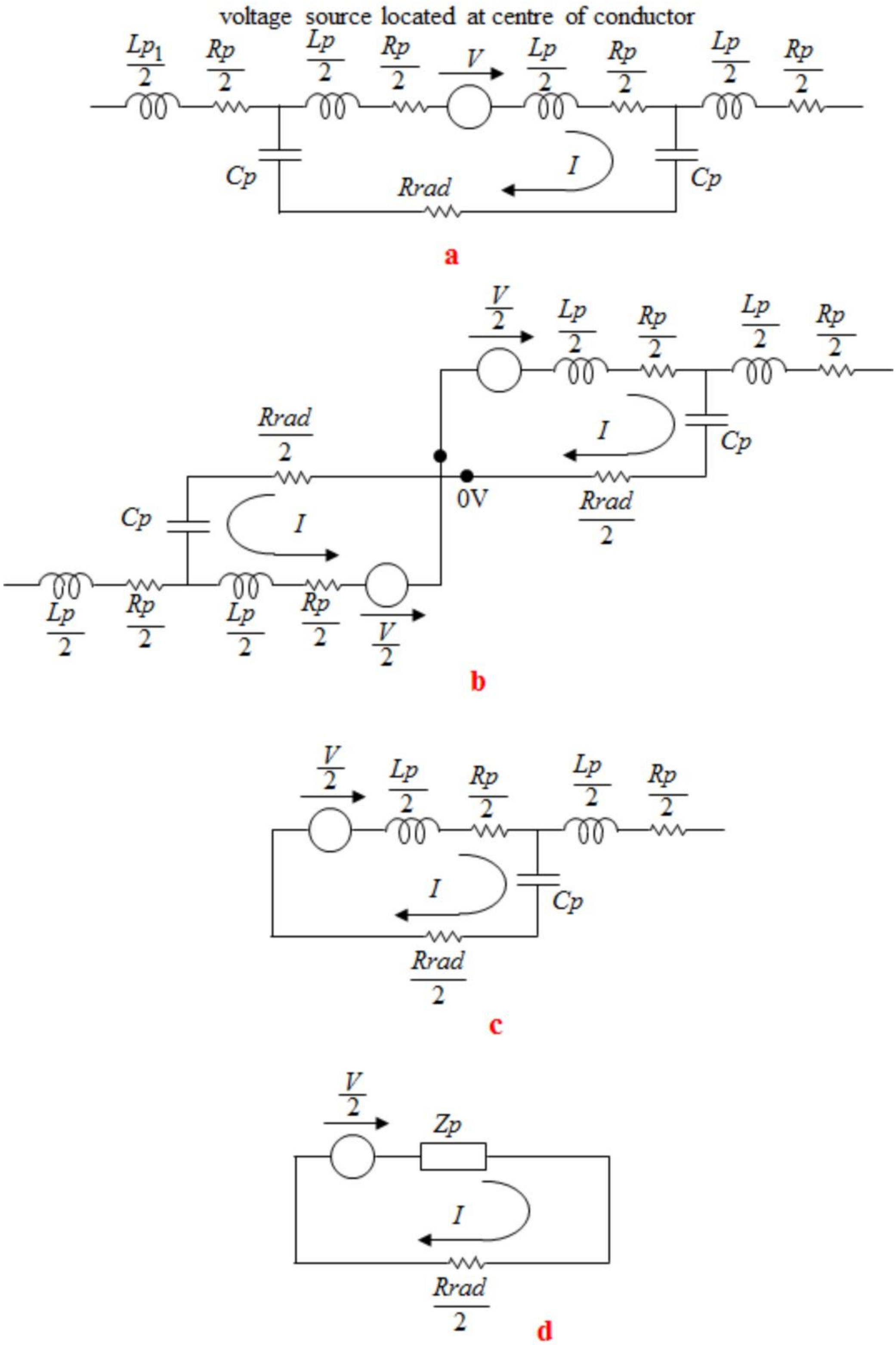

Figure 1 Simplifying the model of a single conductor

a The basic model

b Separating the two sections

c Focussing on the right hand section

d Simulating the reactive components as an impedance

It is then possible to focus on the behaviour of the right hand section, as illustrated by the equivalent circuit of Figure 1c.

If it is assumed that the model is lossless, then it can be simplified further, to that shown by Figure 1d, where

$$Zp = \sqrt{\frac{Lp}{Cp}} \tag{3}$$

The parameter *Zp* correlates with the characteristic impedance *Zo* of a lossless transmission line.

Relating *L* and *C* to *Z*

The article ‘The Transmission Line Model’ describes the derivation of equation (3). It also relates the characteristic impedance of a lossless line to the time *T* taken by the front edge of a signal to propagate along the length *l*.

$$T = \sqrt{Lp \cdot Cp} \tag{4}$$

From equation (3)

$$Cp = \frac{Lp}{Zp^2} \tag{5}$$

Substituting for *Cp* in (4)

$$T = \frac{Lp}{Zp}$$

Hence

$$Lp = T \cdot Zp$$

Substituting for *Lp* in (5)

$$Cp = \frac{T}{Zp}$$

Hence

$$Zp = \frac{T}{Cp} \tag{6}$$

and

$$Zp = \frac{Lp}{T} \tag{7}$$

Twin Conductor Assembly

Taking the model of Figure 1d as a template for each conductor and ignoring the radiation resistance, the simplified model of a twin conductor assembly becomes as Figure 2a. This is not a circuit model. Rather, it is a depiction of the relationships of electromagnetic theory.

$$\begin{aligned} Vp_1 &= Zp_{1,1} \cdot Ip_1 + Zp_{1,2} \cdot Ip_2 \\ Vp_2 &= Zp_{2,1} \cdot Ip_1 + Zp_{2,2} \cdot Ip_2 \end{aligned} \tag{8}$$

Since the impedance parameters in equation (8) are the simplest possible, they are defined as 'primitives'. They relate the voltage along each conductor to the energy level of the electromagnetic field created by the currents in both conductors.

In electrical systems, the currents and voltages monitored by the test equipment are the loop currents and loop voltages, and the relationship between Primitive parameters and Loop parameters is as illustrated, also by Figure 2a.

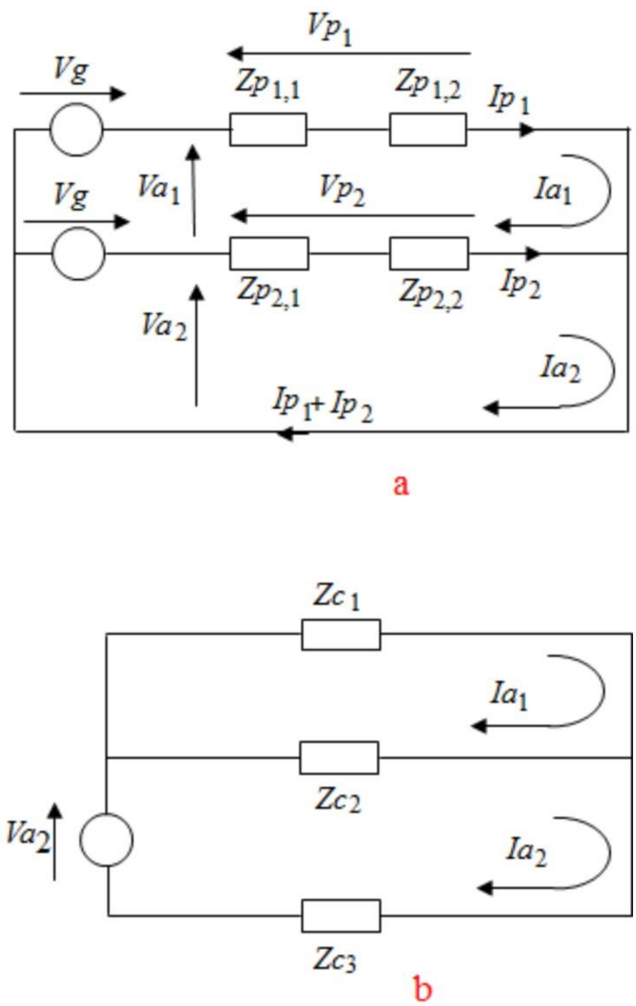

Figure 2 Creating a circuit model of a twin conductor assembly

a Diagrammatic representation of the primitive equations

b Circuit model derived from the primitive equations

The relationship between the currents is defined as:

$$\begin{aligned} Ip_1 &= Ia_1 \\ Ip_2 &= Ia_2 - Ia_1 \end{aligned} \tag{9}$$

The relationship between the voltages is

$$\begin{aligned} Va_1 &= Vp_1 - Vp_2 \\ Va_2 &= Vp_2 \end{aligned} \tag{10}$$

Using (9) to substitute for Ip_1 and Ip_2 in (8)

$$\begin{aligned} Vp_1 &= Zp_{1,1} \cdot Ia_1 + Zp_{1,2} \cdot \left(Ia_2 - Ia_1 \right) \\ Vp_2 &= Zp_{2,1} \cdot Ia_1 + Zp_{2,2} \cdot \left(Ia_2 - Ia_1 \right) \end{aligned}$$

Rearranging:

$$\begin{aligned} Vp_1 &= \left(Zp_{1,1} - Zp_{1,2} \right) \cdot Ia_1 + Zp_{1,2} \cdot Ia_2 \\ Vp_2 &= \left(Zp_{2,1} - Zp_{2,2} \right) \cdot Ia_1 + Zp_{2,2} \cdot Ia_2 \end{aligned}$$

Using (10) to determine the loop voltages

$$\begin{aligned} Va_1 &= \left(Zp_{1,1} - Zp_{1,2} - Zp_{2,1} + Zp_{2,2} \right) \cdot Ia_1 + \left(Zp_{1,2} - Zp_{2,2} \right) \cdot Ia_2 \\ Va_2 &= \left(Zp_{2,1} - Zp_{2,2} \right) \cdot Ia_1 + Zp_{2,2} \cdot Ia_2 \end{aligned} \tag{11}$$

Defining the loop impedances as

$$\begin{aligned} Za_{1,1} &= Zp_{1,1} - Zp_{1,2} - Zp_{2,1} + Zp_{2,2} \\ Za_{1,2} &= Zp_{1,2} - Zp_{2,2} \\ Za_{2,1} &= Zp_{2,1} - Zp_{2,2} \\ Za_{2,2} &= Zp_{2,2} \end{aligned} \tag{12}$$

leads to the loop equations:

$$\begin{aligned} Va_1 &= Za_{1,1} \cdot Ia_1 + Za_{1,2} \cdot Ia_2 \\ Va_2 &= Za_{2,1} \cdot Ia_1 + Za_{2,2} \cdot Ia_2 \end{aligned} \tag{13}$$

For a two-conductor cable:

$$Zp_{1,2} = Zp_{2,1}$$

So, from (12)

$$Za_{1,2} = Za_{2,1} \tag{14}$$

Equation (13) is similar, but not identical, to the equations of Circuit Theory. At this point it was necessary to apply lateral thinking and postulate the existence of a circuit model which can be used to create a similar pair of equations. This task does not take long. The mesh equations for Figure 2b are:

$$\begin{aligned} Va_1 &= \left(Zc_1 + Zc_2\right) \cdot Ia_1 - Zc_2 \cdot Ia_2 \\ Va_2 &= -Zc_2 \cdot Ia_1 + \left(Zc_2 + Zc_3\right) \cdot Ia_2 \end{aligned} \tag{15}$$

Correlating equations (13) and (15) enables loop impedances to be related to the circuit impedances:

$$\begin{aligned} Za_{1,1} &= Zc_1 + Zc_2 \\ Za_{1,2} &= -Zc_2 \\ Za_{2,2} &= Zc_2 + Zc_3 \end{aligned}$$

Defining Circuit Impedances in terms of Loop Impedances

$$\begin{aligned} Zc_1 &= Za_{1,1} + Za_{1,2} \\ Zc_2 &= -Za_{1,2} \\ Zc_3 &= Za_{2,2} + Za_{1,2} \end{aligned} \tag{16}$$

Using equation (12) to substitute primitive impedances for loop impedances:

$$\begin{aligned} Zc_1 &= \left(Zp_{1,1} - Zp_{1,2} - Zp_{2,1} + Zp_{2,2}\right) + \left(Zp_{1,2} - Zp_{2,2}\right) \\ Zc_2 &= Zp_{2,2} - Zp_{1,2} \\ Zc_3 &= Zp_{2,2} + \left(Zp_{1,2} - Zp_{2,2}\right) \end{aligned}$$

This leads to

$$\begin{aligned} Zc_1 &= Zp_{1,1} - Zp_{1,2} \\ Zc_2 &= Zp_{2,2} - Zp_{1,2} \\ Zc_3 &= Zp_{1,2} \end{aligned} \tag{17}$$

Circuit parameters

Using (7) to substitute for *Zp* and *Zc* in (17)

$$\begin{aligned} Lc_1 &= Lp_{1,1} - Lp_{1,2} \\ Lc_2 &= Lp_{2,2} - Lp_{1,2} \\ Lc_3 &= Lp_{1,2} \end{aligned} \tag{18}$$

For two or more conductors,

$$Lp_{i,j} = \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{l}{r_{i,j}}\right) \tag{19}$$

and

$$Cp_{i,j} = \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{l}{r_{i,j}}\right)} \tag{20}$$

where $r_{i,j}$ is the separation between the centres of conductors *i* and *j* and $r_{i,i}$ is the radius of conductor *i*. Using (19) to substitute for the primitive parameters in (18)

$$\begin{aligned} Lc_1 &= \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{r_{1,2}}{r_{1,1}}\right) \\ Lc_2 &= \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{r_{1,2}}{r_{2,2}}\right) \\ Lc_3 &= \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{l}{r_{1,2}}\right) \end{aligned} \tag{21}$$

Using (6) to substitute for the primitive parameters in (18) and rearranging

$$\begin{aligned} \frac{1}{Cc_1} &= \frac{1}{Cp_{1,1}} - \frac{1}{Cp_{1,2}} \\ \frac{1}{Cc_2} &= \frac{1}{Cp_{2,2}} - \frac{1}{Cp_{1,2}} \\ \frac{1}{Cc_3} &= \frac{1}{Cp_{1,2}} \end{aligned} \tag{22}$$

Using (20) to substitute for the primitive parameters in (22)

$$\begin{aligned} Cc_1 &= \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{r_{1,2}}{r_{1,1}}\right)} \\ Cc_2 &= \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{r_{1,2}}{r_{2,2}}\right)} \\ Cc_1 &= \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{l}{r_{1,2}}\right)} \end{aligned} \tag{23}$$

The Virtual Conductor

Replacing each Z-parameter of Figure 2b with the T-network of Lc and Cc parameters, adding the resistors Rc to cater for the fact that the conductors possess series resistance, and incorporating the left hand section into the model, leads to the complete circuit model: Figure 3. The voltage sources can be created by a transformer coupled round both conductors.

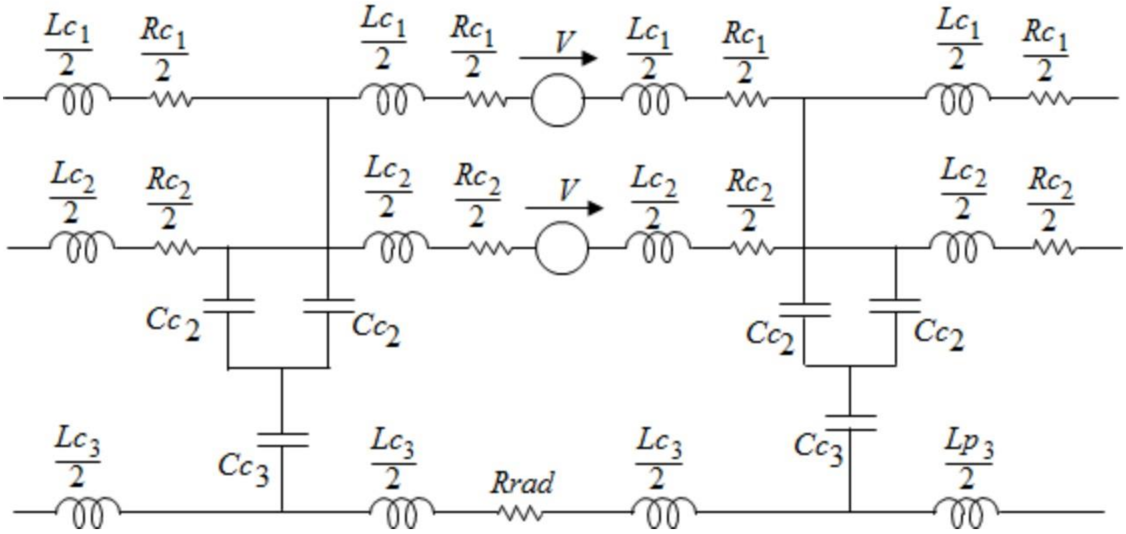

Figure 3 Complete circuit model of a twin conductor assembly

The striking feature of this model is that, for each monopole, it includes a T-network of L, C and R components: arrayed in exactly the same way as those representing an actual conductor. A value can be assigned to every one of them. Since this conductor does not exist in the real world, the most appropriate name must be the 'Virtual Conductor'

There are two current loops involved; the Differential Mode loop which carries signal (or power) current along the send conductor and back via the return conductor; and the Antenna-Mode loop which carries current along both conductors and out into the environment.

A significant feature of this model is that the parameters assigned to each conductor are exactly the same as those derived for a twin-conductor assembly by traditional analytical techniques, where it is assumed that Antenna-Mode current does not exist. The reason for this is that partial currents flow in both directions along every conductor. If there is any imbalance in the forward flowing currents (or the backward-flowing currents), then the only place for the unbalanced current to flow is out into the environment.

The existence of partial currents can be catered for in circuit models, since the Principle of Superposition allows for just such an eventuality.

It is useful to note that the effective radius of the Virtual Conductor is the separation between the two actual conductors; $r_{1,2}$.