Updating Circuit Theory - The Nature of Shielding

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Introduction

The screen of a coaxial cable does not form a barrier to electromagnetic interference. It neutralises the effect of that interference.

Transient current flows along the core of a coaxial cable and creates an electromagnetic field which radiates out through the screen into the environment. In doing so, it induces a voltage in the screen which causes a current to flow in the opposite direction to that in the core. The screen current creates one field which radiates into the environment and another which radiates energy back to the core. The fields emanating from the core and the screen into the environment neutralise each other.

If a third conductor is routed alongside the cable and a step voltage applied between this conductor and the screen, current will flow along this conductor and back via the screen. The field emanating from the screen will induce a voltage along the core. But the field emanating from the third conductor will create an equal and opposite voltage.

Such a setup is analysed using the technique of composite conductors. The analysis shows that the shield behaves in the manner described above.

This type of analysis represents the surface of each conductor as a set of elemental conductors. Voltage induced in any conductor creates current in that conductor. Current in any conductor creates a voltage in every other conductor. It is assumed that the electromagnetic field penetrates conducting material.

The analysis is based on the relationships of Electromagnetic Theory and the results are in line with observations of the behaviour of actual circuit configurations.

Section under review.

This analysis invokes the concept of composite conductors, where each composite is represented by a set of elemental conductors arrayed round its periphery. The circumference of each elemental conductor is defined to be the same as the path length of the section of the surface it represents. This ensures that the strength of the magnetic field on the surface of the element is identical to that on the surface of the surface it simulates.

The section under review is that of a co-axial cable routed alongside a second conductor. The radius *R1* of the core of the coax is 0.33 mm. The radius *R2* of the screen is 1.9 mm, as is the radius *R3* of the third conductor. The separation *sep* between screen and third conductor is 6 mm. Each of these conductors is represented as an array of twelve elemental conductors. So there are three composite conductors and 36 elemental conductors. Figure 1 is a copy of the first page of a Mathcad worksheet. The cross-section is illustrated at the bottom of the figure.

$X1 := 3 \cdot 10^{-3}$ $Y1 := 3 \cdot 10^{-3}$ $R1 := 0.33 \cdot 10^{-3}$ $n_1 := 12$

$X2 := X1$ $Y2 := Y1$ $R2 := 1.9 \cdot 10^{-3}$ $n_2 := 12$

$X3 := 9 \cdot 10^{-3}$ $Y3 := 3 \cdot 10^{-3}$ $R3 := 1.9 \cdot 10^{-3}$ $n_3 := 12$

$N := n_1 + n_2 + n_3 = 36$ $sep := X3 - X1 = 6 \times 10^{-3}$

$i := 1 .. n_1$

$\theta_i := \frac{2\pi}{n_1} \cdot (i)$

$x_i := X1 + R1 \cdot \cos(\theta_i)$ $y_i := Y1 + R1 \cdot \sin(\theta_i)$ $r_i := \frac{R1}{n_1}$

$i := n_1 + 1 .. n_1 + n_2$

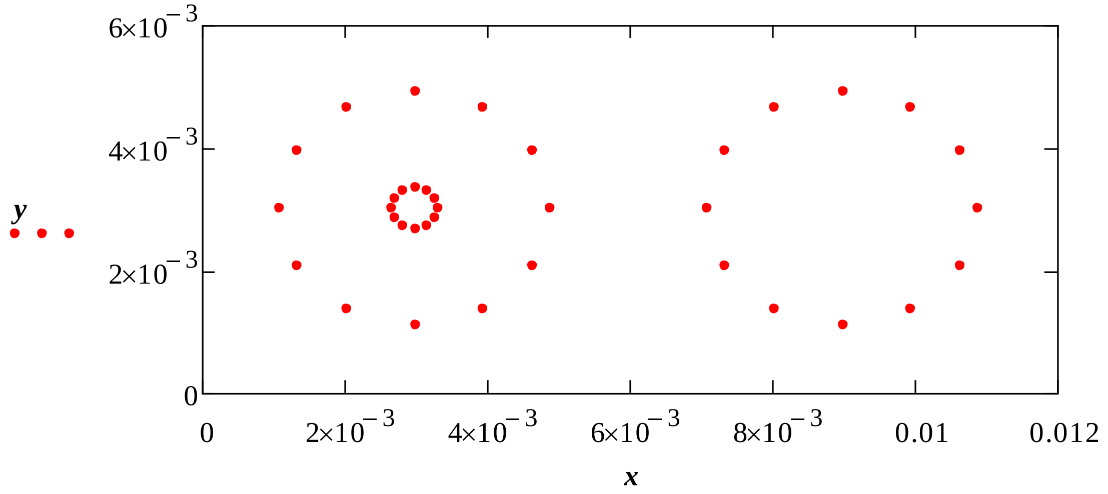
$\theta_i := \frac{2\pi}{n_2} \cdot (i - n_1)$

$x_i := X2 + R2 \cdot \cos(\theta_i)$ $y_i := Y2 + R2 \cdot \sin(\theta_i)$ $r_i := \frac{R2}{n_2}$

$i := n_1 + n_2 + 1 .. N$

$\theta_i := \frac{2\pi}{n_3} \cdot (i - n_1 - n_2)$

$x_i := X3 + R3 \cdot \cos(\theta_i)$ $y_i := Y3 + R3 \cdot \sin(\theta_i)$ $r_i := \frac{R3}{n_3}$



$i := 1 .. N$ $A_{i,1} := x_i \cdot 10^3$ $A_{i,2} := y_i \cdot 10^3$ $A_{i,3} := r_i \cdot 10^3$

Figure 1 Defining the cross section of the cable assembly

Primitive Currents

Figure 2 is a copy of the second page of the worksheet. This calculates the amplitude of the current flowing in every elemental conductor when a 1V step is applied between the core and screen at one end of the co-axial cable; that is, between composite conductors 1 and 2.

The basic equation used in this computation defines the impedance values assigned to the elemental conductors:

$$Zp_{i,j} = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{\mu}{\varepsilon}} \cdot \ln\left(\frac{1}{r_{i,j}}\right) \quad \text{ohm/metre} \qquad (1)$$

where μ is the permeability of the conductors, ε is the permittivity of the insulation, *i* and *j* identify the elemental conductors, and $r_{i,j}$ defines the radial separation between these conductors. If *i* is not equal to *j*, then $Zp_{i,j}$ is the mutual impedance. If *i* equals *j*, then $Zp_{i,i}$ defines the self-impedance of conductor *i* and $r_{i,i}$ is its radius. Equation (1) is derived in the article ‘The Three Conductor Model’

The loop impedance $Zloop_{i,j}$ is the impedance due to current flowing down one conductor and back via the adjacent conductor. The vector *Vloop* is defined as a set of voltage sources between each pair of elemental conductors, where the voltage between elemental conductors 12 and 13 is set at 1 Volt and all other voltages set at zero. This simulates a voltage of 1 volt applied to composite conductor 1 with respect to composite conductor 2.

The function *lsolve*(*Zloop*,*Vloop*) calculates the value of the current *Iloop* in every loop between adjacent conductors. When these values are known, it is a simple matter to assign a value to the current *Ip* in every elemental conductor. The final computation illustrated in Figure 2 creates a table of values of the co-ordinates of each elemental conductor and the current it carries. This table was copied to Microsoft Excel software, which was used to create a bubble plot of the results.

This bubble plot is shown by Figure 3. The size of each bubble is proportional to the amplitude of the current in that particular elemental conductor. Co-ordinates for conductor 3 do not appear on this diagram for the simple reason that the total current flow is infinitesimal.

This figure illustrates the fact that current flows forward along the core of the co-axial cable and back via the screen. Since no current flows on conductor 3, it would appear that the screen acts as an impenetrable shield.

It could be reasoned that this is because current only flows along the inner surface of the shield. But this does not apply to braided shields where each strand is exposed to the outer environment as well as to the central core. Nor does it apply to a solid shield, where the field inside the conductor reverberates between inner and outer surfaces. With a solid shield, it would be expected that high frequency current flows on both these surfaces.

$\mu := 4\cdot\pi\cdot 10^{-7}$ $\varepsilon := 8.854\cdot 10^{-12}$ Fundamental constants

$$Zp := \begin{array}{|l} H \leftarrow \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{\mu}{\varepsilon}} \\ \text{for } i \in 1..N \\ \quad \text{for } j \in 1..N \\ \quad\quad \begin{array}{|l} h \leftarrow \mathbf{x}_j - \mathbf{x}_i \\ v \leftarrow \mathbf{y}_j - \mathbf{y}_i \\ rad \leftarrow \sqrt{h^2 + v^2} \\ rad \leftarrow \mathbf{r}_i \text{ if } rad = 0 \\ \mathbf{Z}_{i,j} \leftarrow H\cdot ln\left(\frac{1}{rad}\right) \end{array} \\ \mathbf{Z} \end{array}$$

Calculating the primitive impedance of every elemental conductor.

The first action is to calculate the separation between every conductor and every other conductor, *rad*. Then this value is used to calculate the value of the primitive inductance

Then the results are stored in an array of N rows by N columns

$$Zloop := \begin{array}{|l} \text{for } i \in 1..N-1 \\ \quad \text{for } j \in 1..N-1 \\ \quad\quad \mathbf{Z}_{i,j} \leftarrow \mathbf{Zp}_{i,j} - \mathbf{Zp}_{i,j+1} - \mathbf{Zp}_{i+1,j} + \mathbf{Zp}_{i+1,j+1} \\ \mathbf{Z} \end{array}$$

Calculating the loop impedance of each adjacent pair of elemental conductors

$$Vloop := \begin{array}{|l} \text{for } i \in 1..N-1 \\ \quad \begin{array}{|l} \mathbf{V}_i \leftarrow 0 \\ \mathbf{V}_i \leftarrow 1 \text{ if } i = \mathbf{n}_1 \end{array} \\ \mathbf{V} \end{array}$$

Locating the voltage source in loop $\mathbf{n}_1$

$$Iloop := lsolve(\mathbf{Zloop}, \mathbf{Vloop})$$

calculating the values of all the loop currents.

$$Ip := \begin{array}{|l} \mathbf{I}_1 \leftarrow \mathbf{Iloop}_1 \\ \text{for } i \in 2..N-1 \\ \quad \mathbf{I}_i \leftarrow \mathbf{Iloop}_i - \mathbf{Iloop}_{i-1} \\ \mathbf{I}_N \leftarrow -\mathbf{Iloop}_{N-1} \\ \mathbf{I} \end{array}$$

Using data on the values of the loop currents to derive values for the primitive currents.

$\mathbf{B}_{i,1} := \mathbf{x}_i\cdot 10^3$ $\mathbf{B}_{i,2} := \mathbf{y}_i\cdot 10^3$ $\mathbf{B}_{i,3} := \mathbf{Ip}_i\cdot 10^3$ Tabulating the results

Figure 2 Calculating the amplitude of the current in each elemental conductor

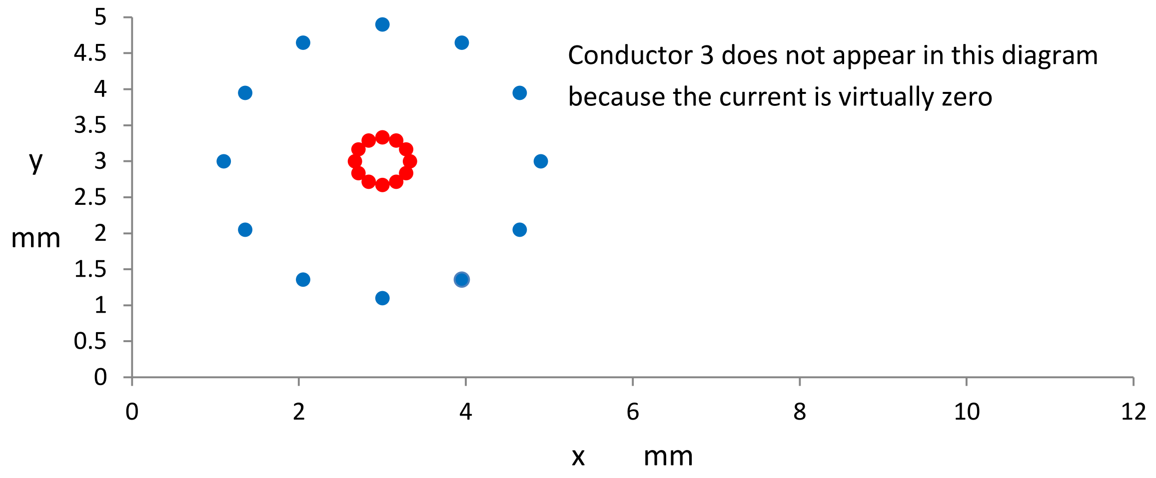


Figure 3 Bubble plot of current in each elemental conductor

The current which flows along the screen creates an electromagnetic field along the outer surface of the coaxial cable. Since this field radiates out into the environment, it is certain to create a voltage along the length of composite conductor 3. But the total current along this conductor is effectively zero. This can be explained by calculating all twelve partial voltages in the composite conductors of the assembly.

Partial Voltages

The worksheet illustrated by Figure 4 performs the relevant computations.

The vectors ***Start*** and ***End*** at the top of the page define the first and last elemental conductors of those which simulate composite conductors 1, 2 and 3. These are used to calculate the total current flow along each composite. The results are stored in the vector ***Iq***. A current of 9.527mA flows down the core conductor and returns via the screen. The current in the third composite conductor 3 is calculated to be 2.96 nA. In practical terms, the total voltage applied to this conductor by the electromagnetic field is zero

The next Mathcad function calculates the voltage contributions due to mutual coupling and self-coupling and stores the results in the vector ***vq***.

The partial voltages developed along conductor 3 are

due to self-coupling $\boldsymbol{vq}_{3,3}$ = 1 micro-Volt

due to current in conductor 1 $\boldsymbol{vq}_{3,1}$ = 2.932 Volt

due to current in conductor 2 $\boldsymbol{vq}_{3,2}$ = -2.932 Volt

The partial voltages due to mutual coupling are generated by currents flowing in opposite directions in every elemental conductor in composite conductor 3. Since the forward current flow is identical to the return current flow, the nett current flow is infinitesimal, and so is the

$$\boldsymbol{Start} := \begin{pmatrix} 1 \\ \boldsymbol{n}_1 + 1 \\ \boldsymbol{n}_1 + \boldsymbol{n}_2 + 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 13 \\ 25 \end{pmatrix}$$

defining the first elemental conductor in each composite conductor

$$\boldsymbol{End} := \begin{pmatrix} \boldsymbol{n}_1 \\ \boldsymbol{n}_1 + \boldsymbol{n}_2 \\ N \end{pmatrix} = \begin{pmatrix} 12 \\ 24 \\ 36 \end{pmatrix}$$

defining the last elemental conductor in each composite conductor

$$\boldsymbol{Iq} := \left| \begin{array}{l} \text{for } h \in 1..3 \\ \quad \left| \begin{array}{l} S \leftarrow 0 \\ \text{for } i \in \boldsymbol{Start}_h .. \boldsymbol{End}_h \\ \quad S \leftarrow S + \boldsymbol{Ip}_i \\ \boldsymbol{I}_h \leftarrow S \end{array} \right. \\ \boldsymbol{I} \end{array} \right.$$

Calculating the total current flowing along each composite conductor

$$\boldsymbol{Iq} \cdot 10^3 = \begin{pmatrix} 9.527 \\ -9.527 \\ 2.96 \times 10^{-6} \end{pmatrix}$$

$$(\boldsymbol{Iq}_1 + \boldsymbol{Iq}_2) \cdot 10^3 = -2.96 \times 10^{-6}$$

$$\boldsymbol{vq} := \left| \begin{array}{l} \text{for } h \in 1..3 \\ \quad \text{for } k \in 1..3 \\ \quad\quad \left| \begin{array}{l} S \leftarrow 0 \\ \text{for } i \in \boldsymbol{Start}_h .. \boldsymbol{End}_h \\ \quad \text{for } j \in \boldsymbol{Start}_k .. \boldsymbol{End}_k \\ \quad\quad S \leftarrow S + \boldsymbol{Zp}_{i,j} \cdot \boldsymbol{Ip}_j \\ V_{h,k} \leftarrow \dfrac{S}{\boldsymbol{n}_h} \end{array} \right. \\ V \end{array} \right.$$

calculating the contribution made to the overall voltage along each conductor by current in each conductor.

$$\boldsymbol{vq} = \begin{pmatrix} 4.579 & -3.579 & 1.136 \times 10^{-6} \\ 3.579 & -3.579 & 1.137 \times 10^{-6} \\ 2.923 & -2.923 & 1.112 \times 10^{-6} \end{pmatrix}$$

$$\boldsymbol{Vq} := \left| \begin{array}{l} \text{for } h \in 1..3 \\ \quad \left| \begin{array}{l} S \leftarrow 0 \\ \text{for } k \in 1..3 \\ \quad S \leftarrow S + \boldsymbol{vq}_{h,k} \\ V_h \leftarrow S \end{array} \right. \\ V \end{array} \right.$$

calculating the voltage developed along every composite conductor

$$\boldsymbol{Vq} = \begin{pmatrix} 1 \\ 2.437 \times 10^{-8} \\ 2.437 \times 10^{-8} \end{pmatrix}$$

Figure 4 Computing the partial voltages developed along each conductor.

total voltage. The vector ***Vq*** demonstrates this. The total voltage induced in conductor 3 is calculated to be $\boldsymbol{Vq_3}$ = 24 nV.

Since current is flowing in both directions in the shield, two opposing electromagnetic waves emanate from the outer surface. These induce two voltages in conductor 3. Since these are equal in magnitude but opposite in direction, they have minimal effect on that conductor.

Voltages along elemental conductors

The effect can be illustrated in detail by calculating the value of each partial voltage in each elemental conductor of composite 3 and tabulating the results. This is recorded by Table 1

Conductor 3	Voltage due to current in:		
element	Core	Screen	Conductor 3
25	2.78	-2.78	9.37E-07
26	2.823	-2.823	9.47E-07
27	2.895	-2.895	9.58E-07
28	2.992	-2.992	8.55E-07
29	3.092	-3.092	-3.76E-07
30	3.14	-3.14	5.77E-06
31	3.092	-3.092	-3.76E-07
32	2.992	-2.992	8.55E-07
33	2.895	-2.895	9.58E-07
34	2.823	-2.823	9.47E-07
35	2.78	-2.78	9.37E-07
36	2.765	-2.765	9.33E-07

Table 1 Partial voltages in the elemental conductors of composite conductor 3

This table shows that the current in the coaxial core of the screened cable creates a positive voltage in series with every elemental conductor of composite conductor 3, whilst current on the cable screen creates a negative voltage in series with every elemental conductor. The nett voltage induced in every elemental conductor is negligible. Since the nett current flow along every elemental conductor of composite 3 is virtually zero, then so is the partial voltage due to that current.

The three partial voltages recorded on the bottom line of the array ***vq*** in Figure 4 are the average values of the partial voltages along the elemental conductors of composite 3

Component values for coaxial cable

Having calculated the values of the currents flowing in each of the composite conductors and the voltage developed along each conductor, it is possible to assign values to the circuit impedances. The computations involved are illustrated by Figure 5, which is a copy of the final page of the worksheet.

There are two loops involved. In loop 1, current flows along conductor 1 and flows back along conductor 2. In loop 2, current flows along conductor 2 and returns via conductor 3. The driving force is a voltage of 1Volt, applied between the terminations of conductors 1 and 2 at the near-end of the cable assembly. The voltage applied to loop 2 is zero.

The function at the top of Figure 5 calculates to values of the loop currents and stores them in the vector ***Iloop***. The total voltage developed in each loop is calculated by adding the voltages developed along send and return conductors. The minus sign in the function ***Vc*** is because current flowing back via the return conductor is defined as negative.

$$\boldsymbol{Iloop} := \left| \begin{array}{l} I_1 \leftarrow \boldsymbol{Iq}_1 \\ I_2 \leftarrow -\boldsymbol{Iq}_3 \\ I \end{array} \right. \qquad \boldsymbol{Iloop} = \begin{pmatrix} 9.527 \times 10^{-3} \\ -2.96 \times 10^{-9} \end{pmatrix}$$ Loop currents

$$\boldsymbol{Vloop} := \left| \begin{array}{l} V_1 \leftarrow \boldsymbol{Vq}_1 - \boldsymbol{Vq}_2 \\ V_2 \leftarrow \boldsymbol{Vq}_2 - \boldsymbol{Vq}_3 \\ V \end{array} \right. \qquad \boldsymbol{Vloop} = \begin{pmatrix} 1 \\ -2.187 \times 10^{-15} \end{pmatrix}$$ Loop voltages

$$T := \sqrt{\mu \cdot \varepsilon} = 3.336 \times 10^{-9}$$ Time for step to propagate along one metre

$$Zc1 := \frac{\boldsymbol{Vloop}_1}{\boldsymbol{Iloop}_1} = 104.959$$ Impedance of loop 1, per metre

$$Lc1 := T \cdot Zc1 = 3.501 \times 10^{-7}$$ Inductance of loop 1, per metre

$$\frac{Lc1}{2} = 1.751 \times 10^{-7}$$ component of circuit model

$$Cc1 := \frac{T}{Zc1} = 3.178 \times 10^{-11}$$ component of circuit model

$$Ctheory := \frac{2 \cdot \pi \cdot \varepsilon}{ln\left(\frac{R2}{R1}\right)} = 3.178 \times 10^{-11}$$ Capacitance derived from textbook theory

Figure 5 Calculating the value of the capacitance between composite conductors 1 and 2

Since the voltage developed in loop 2 is $\boldsymbol{Vc}_2 = 2 \times 10^{-15}$, there is no point in calculating the impedance of that loop. But there is no problem in assigning a value of 105 ohm to the impedance of loop 1, and this leads to the derivation of a value of 350nH for the inductance of loop 1 and 32pF for the capacitance between conductors 1 and 2. The fact that the value of the capacitance is exactly the same as that derived from textbook theory provides a high degree of confidence in the modelling technique.

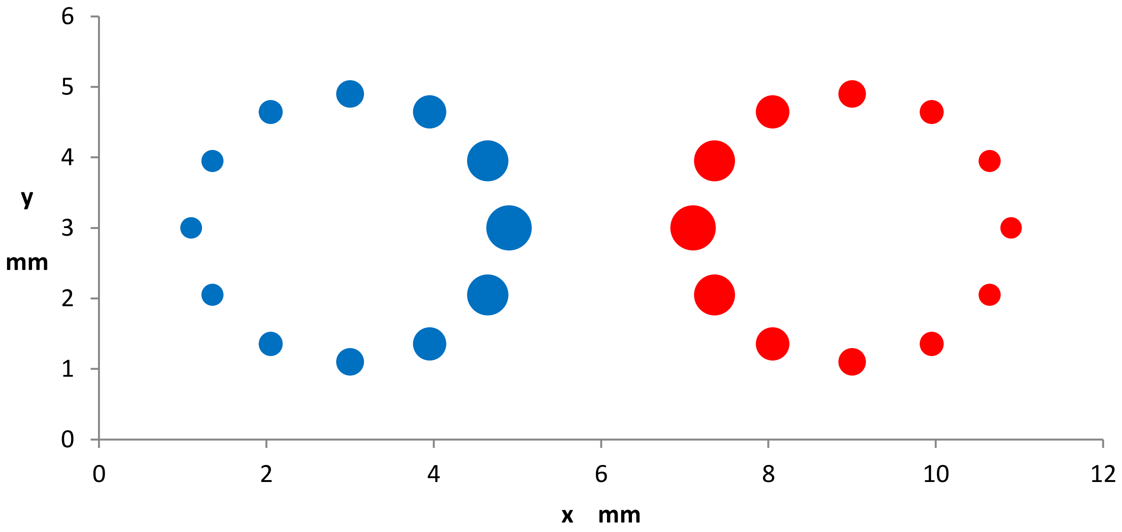
Susceptibility

External interference can be simulated by removing the voltage source from loop 1 and inserting it in loop 2. The simulation involves copying the worksheet to another file and then modifying the copy. The only change to the main computation is the definition of the function ***Vloop***. This modification is illustrated by Figure 6. The voltage on elemental conductor 24 is defined as -1 volt with respect to elemental conductor 25. So the voltage on conductor 25 is positive with respect to conductor 24. The resultant current distribution is shown by the bubble chart of Figure 7. The red circles indicate that the current is flowing forward. Blue circles indicate that current is flowing from the far end to the near end of the assembly

$$\boldsymbol{Vloop} := \left| \begin{array}{l} \textit{for}\ \ i \in 1 .. N - 1 \\ \quad \left| \begin{array}{l} \boldsymbol{V}_i \leftarrow 0 \\ \boldsymbol{V}_i \leftarrow -1 \ \ \textit{if}\ \ i = \boldsymbol{n}_1 + \boldsymbol{n}_2 \end{array} \right. \\ \boldsymbol{V} \end{array} \right.$$

Locating the voltage source in loop 2

Figure 6 Locating the voltage source between composite conductors 2 and 3



note: conductor 1 does not appear, because it carries zero current

Figure 7 Current distribution when 1 Volt is applied between conductor 3 and conductor 2

The partial voltages along each conductor are computed on the third page of the revised worksheet, illustrated by figure 8. This shows that the voltages induced in conductor 1 are identical to those in conductor 2. That is, external electromagnetic interference penetrates the

```
vq :=  for h ∈ 1..3
         for k ∈ 1..3
             S ← 0
             for i ∈ Start_h .. End_h
                for j ∈ Start_k .. End_k
                   S ← S + Zp_{i,j} · Ip_j
             V_{h,k} ← S / n_h
       V
```

calculating the contribution made to the overall voltage along each conductor by current in each conductor.

$$\boldsymbol{vq} = \begin{pmatrix} 1.423\times10^{-6} & -3.041 & 2.541 \\ 1.112\times10^{-6} & -3.041 & 2.541 \\ -1.496\times10^{-6} & -2.541 & 3.041 \end{pmatrix}$$

```
Vq :=  for h ∈ 1..3
          S ← 0
          for k ∈ 1..3
             S ← S + vq_{h,k}
          V_h ← S
       V
```

calculating the voltage developed along every composite conductor

$$\boldsymbol{Vq} = \begin{pmatrix} -0.5 \\ -0.5 \\ 0.5 \end{pmatrix}$$

Figure 8 Computing the partial voltages when voltage source is in loop 2

shield and induces a voltage along the core conductor. However, the current in the screen creates a voltage which matches that voltage. From Figure 8, $Vq_1 = Vq_2 = -0.5$ Volt.

4.2 External Components

Calculating the value of the L and C values for composite conductors 2 and 3 involves the same process as that used to calculate that between conductors 1 and 2. Figure 9 illustrates the computations. In this case, the value of $\boldsymbol{Vloop}_1$ is far too small to calculate reliable values for those associated with the loop between composite conductors 1 and 2.

The value derived for *Cc3* can be compared to that derived in textbooks of Electromagnetic Theory for the capacitance between two tubular conductors:

$$Ctheory = \frac{\pi \cdot \varepsilon}{\ln\left(\dfrac{b+\sqrt{b^2-r^2}}{r}\right)} = 2.701\times10^{-11} \text{ Farad per metre} \qquad (2)$$

where $b = \dfrac{sep}{2}$ and $r = R2 = 1.9\times10^{-11}$

$$\boldsymbol{Iloop} := \left| \begin{array}{l} I_1 \leftarrow \boldsymbol{Iq}_1 \\ I_2 \leftarrow -\boldsymbol{Iq}_3 \\ I \end{array} \right. \qquad \boldsymbol{Iloop} = \begin{pmatrix} 2.96 \times 10^{-9} \\ -8.094 \times 10^{-3} \end{pmatrix}$$

Loop currents

$$\boldsymbol{Vloop} := \left| \begin{array}{l} V_1 \leftarrow \boldsymbol{Vq}_1 - \boldsymbol{Vq}_2 \\ V_2 \leftarrow \boldsymbol{Vq}_2 - \boldsymbol{Vq}_3 \\ V \end{array} \right. \qquad \boldsymbol{Vloop} = \begin{pmatrix} 2.665 \times 10^{-15} \\ -1 \end{pmatrix}$$

Loop voltages

$$T := \sqrt{\mu \cdot \varepsilon} = 3.336 \times 10^{-9}$$

Time for step to propagate along one metre

$$Zc3 := \frac{\boldsymbol{Vloop}_2}{\boldsymbol{Iloop}_2} = 123.543$$

Impedance of loop 1, per metre

$$Lc3 := T \cdot Zc3 = 4.121 \times 10^{-7}$$

Inductance of loop 1, per metre

$$\frac{Lc3}{2} = 2.06 \times 10^{-7}$$

component of circuit model

$$Cc3 := \frac{T}{Zc3} = 2.7 \times 10^{-11}$$

component of circuit model

Figure 9 Computing values for inductance and capacitance of external loop

The fact that there is close correlation between *Cc3* and *Ctheory* provides a high level of confidence in the method of composite conductors.

Circuit model

Having computed values for the circuit components of both the internal and external loops, there is now enough information to construct a circuit model of the assembly under review

The values for *Lc1* and *Cc1* are derived from Figure 5, whilst *Lc3* and *Cc3* are derived from Figure 9.

It would seem that the only component coupling energy through the shield is the resistance *Rc2* of the screen. But that is an illusion. The voltage induced in the shield by the interfering source is cancelled out by the self-induced voltage.

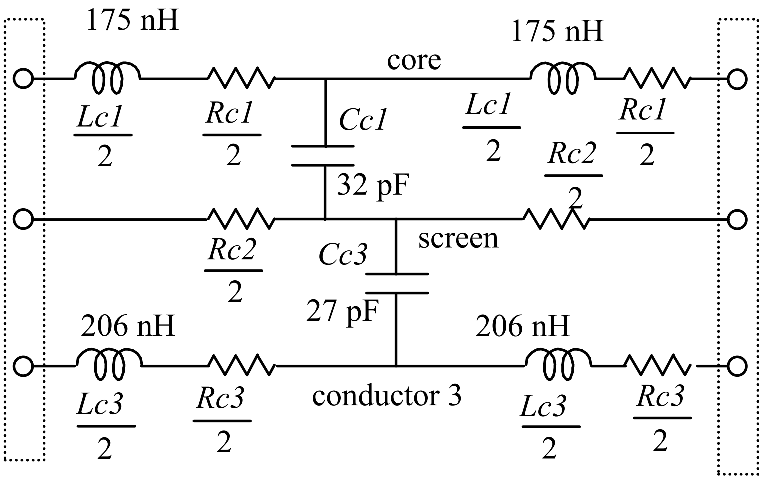


Figure 10 Circuit model of 1 metre length of cable assembly under review

Conclusion

It has been shown that the action of the screen is to neutralise the interference, whether the source of that interference is internal or external to the signal carried by the coaxial cable

Since the analysis has been derived from electromagnetic theory and since it predicts the actual behaviour of the system under review, it can reasonably be concluded that the mechanisms involved in the propagation of EMI are as described in the introduction.

A circuit model has been derived to simulate the coupling between the internal and external loops.

The technique can be developed to simulate the cross coupling between the differential mode and common mode loops of any cable assembly.