Updating Circuit Theory

The Three Conductor Model.

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1 Introduction

By assigning three conductors to any signal or power link between two units of equipment in any electrical system, a circuit model can be created which enables the interference coupling mechanisms of that link to be analysed. This article shows how such a model can be derived from the equations of Electromagnetic Theory.

Electromagnetic Theory leads to the conclusion that current in any conductor will create a voltage in every other conductor via electromagnetic field coupling. Everything interferes with everything else. But that is not the case with Circuit Theory.

Circuit Theory is based on the assumption that the impedance of every component, be that a resistance, an inductance, or a capacitance, is purely a function of the properties of that component. Interconnecting wires are defined as having zero impedance. This includes the conductors of the supporting structure, which is assumed to be equipotential.

Such a set of assumptions is purely fictitious, and the engineers who developed Circuit Theory must have been well aware of that fact. Even so, it was a good working hypothesis at the time, since the power handled by components and devices was much higher that could be delivered by any stray electromagnetic field. That is, a 'don't care' condition applied to the existence of electromagnetic interference (EMI). The justification must have been that it can be used to model the behaviour of complex electric systems.

The problem is that circuit models have proved to be so accurate and reliable that engineers have come to believe that the assumptions are valid. It is widely accepted that the structure can be represented by an equipotential surface. This is evident in the universal use of the ubiquitous 'earth' symbol. The belief that the impedance of a conductor is purely the property of that conductor is evident in the implementation of the 'single-point ground' in the design of many systems.

Given such beliefs, it is not surprising that the analysis of EMI has been declared to be a 'black art'. Circuit theory has been abandoned in favour of esoteric models involving full-field simulation. It has been declared that there are four independent 'types' of interference; common-impedance coupling, inductive coupling, capacitive coupling, and radiation coupling. There is much confusion. The complexity is limitless.

However, if Electromagnetic Theory is reviewed, to identify the links it has with Circuit Theory, it becomes evident that the solution to EMI problems has been there all the time. Equations relating currents and voltages to the behaviour of the electromagnetic field are defined by the former theory. Analytical tools to solve those equations are provided by the latter.

At least three conductors are needed to create a culprit loop and a victim loop. So, an essential requirement for any circuit model which can analyse EMI is that it can simulate the coupling between at least three conductors. The derivation of formulae for the inductance and capacitance of the conductors of a three-phase power line is in all the basic Electrical Engineering textbooks. This article describes how these formulae can be used to create a three-conductor model.

2 The Primitives

2.1 Primitive Capacitance

Figure 1 illustrates the relationship between a length of charged conductor and the electric field at a point. It is assumed that the length of the section is *l* metres, the line charge density is ρ coulombs/metre and that the point *P*(*r*,*z*) is located a radial distance *r* metres from the centre of the conductor. Then the radial component of the electric field is:

$$E = \frac{\rho}{4 \cdot \pi \cdot \varepsilon} \cdot \frac{1}{r} \cdot \left[\sin(\alpha_1) + \sin(\alpha_2) \right] \text{Volt/metre} \qquad (1)$$

The work done on this section of conductor due to unit charge moving from infinity to the surface of the conductor can be calculated by integrating the value of the electric field over the distance between $r = \infty$ and $r = 0$. This leads to:

$$Vp_{1,1} = \frac{\rho}{2 \cdot \pi \cdot \varepsilon} \cdot \ln\left(\frac{l}{r_{1,1}} \right) \text{Volt} \qquad (2)$$

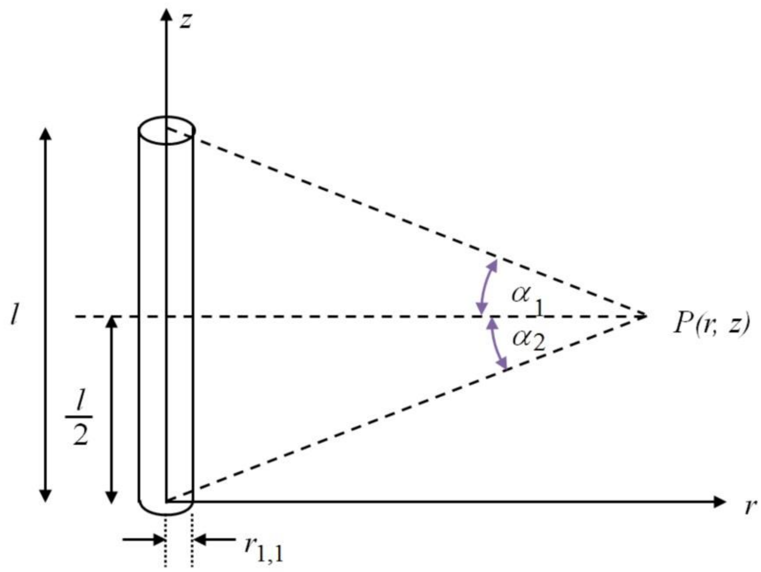

Figure 1 Electric field at a point

The capacitance of the conductor is the ratio of charge to voltage. So:

$$Cp_{1,1} = \frac{\rho \cdot l}{Vp_{1,1}} = \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{l}{r_{1,1}}\right)} \text{ Farad} \tag{3}$$

If a second conductor is installed parallel to conductor 1, as shown on Figure 2, then the voltage on conductor 2 due to the charge on conductor 1 would be

$$Vp_{2,1} = \frac{\rho}{2 \cdot \pi \cdot \varepsilon} \cdot \ln\left(\frac{l}{r_{2,1}}\right) \quad \text{Volt} \tag{4}$$

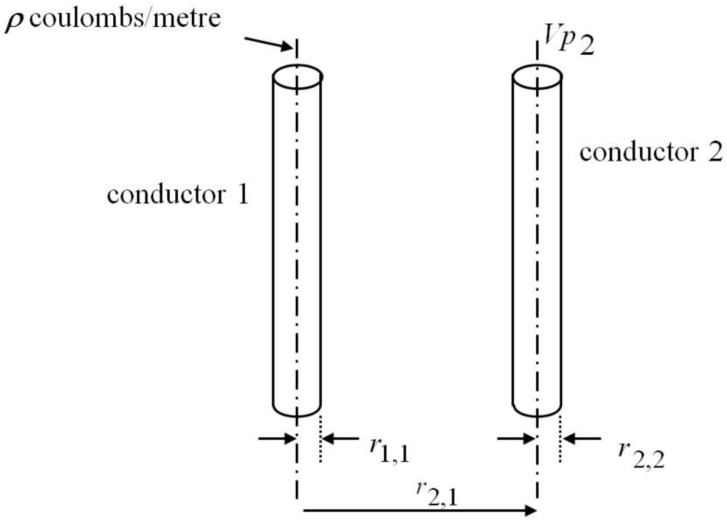

Figure 2 Voltage on conductor 2 due to charge on conductor 1

and the primitive capacitance would be

$$Cp_{2,1} = \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\frac{l}{r_{2,1}}\right)} \text{ Farad} \tag{5}$$

2.2 Primitive Inductance

Figure 3 illustrates a length of conductor with a constant current Ip flowing in the z direction. The magnetic field at the point P is

$$H = \frac{Ip}{4 \cdot \pi \cdot r} \cdot \left[\sin(\alpha_1) + \sin(\alpha_2)\right] \text{ A/m} \tag{6}$$

The flux density B at the point P is:

$$B = \mu \cdot H \text{ Tesla} \tag{7}$$

The total flux φ which passes through a rectangular strip extending from the surface of the conductor to a very large distance is

$$\varphi = \int B \cdot ds \ \text{Wb} \tag{8}$$

where *s* is the surface of the rectangular strip.

Dividing this flux by the current *Ip* gives the primitive inductance:

$$Lp_{1,1} = \frac{\varphi}{Ip} = \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{l}{r_{1,1}}\right) \text{ Henry} \tag{9}$$

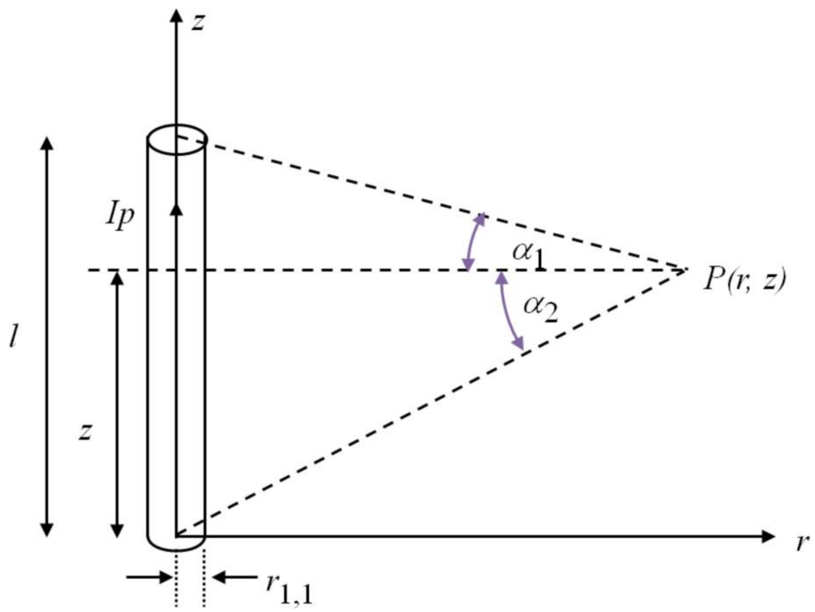

Figure 3 Magnetic field at a point

If a second conductor is routed alongside conductor 1, as illustrated by Figure 4, the primitive inductance of conductor 2 can be defined as

$$Lp_{2,1} = \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{l}{r_{2,1}}\right) \text{ Henry} \tag{10}$$

There are a lot of integrations between equations (1) and (2), and between equations (6) and (10). The full derivation can be found at http://www.designemc.info/21Primitive.pdf

The purpose of this reprise of textbook theory is to highlight the fact that all the activity occurs outside the conductors. It is clear from the derivation of Primitive Capacitance and Primitive Inductance that they are properties of the electromagnetic field in the region surrounding the conductors. They are not the unique properties of the conductors. Even so, these parameters are two of the building blocks for all circuit models.

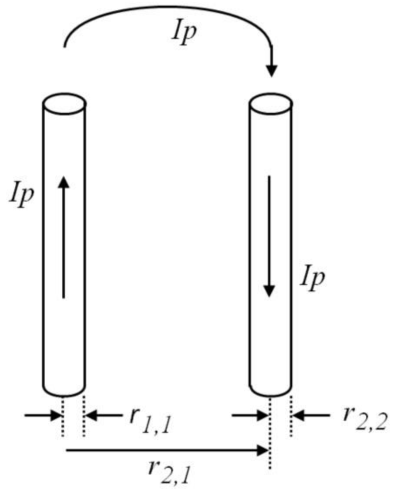

Figure 4 Current in a two-conductor assembly

The term ‘Primitive’ has been used because these are the simplest formulae which can be assigned the dimensions of Farad and Henry.

2.3 Primitive Impedance

Transmission line theory derives an expression for the characteristic impedance of a zero-loss line. For a single, isolated, conductor, this would be

$$Zp_{1,1} = \sqrt{\frac{Lp_{1,1}}{Cp_{1,1}}} \tag{11}$$

Invoking (9) and (3) to substitute for Lp_{11} and Cp_{11}

$$Zp_{1,1} = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{\mu}{\varepsilon}} \cdot \ln\left(\frac{l}{r_{1,1}}\right) \tag{12}$$

where $r_{1,1}$ is the radius of the conductor and l is the length.

2.4 Primitive Equations

For a three-conductor cable where the cross section is as defined by Figure 5, the Primitive Equations are:

$$\begin{aligned}
Vp_1 &= Zp_{1,1} \cdot Ip_1 + Zp_{1,2} \cdot Ip_2 + Zp_{1,3} \cdot Ip_3 \\
Vp_2 &= Zp_{2,1} \cdot Ip_1 + Zp_{2,2} \cdot Ip_2 + Zp_{2,3} \cdot Ip_3 \\
Vp_3 &= Zp_{3,1} \cdot Ip_1 + Zp_{3,2} \cdot Ip_2 + Zp_{3,3} \cdot Ip_3
\end{aligned} \tag{13}$$

Where

$$Zp_{i,j} = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{\mu}{\varepsilon}} \cdot \ln\left(\frac{l}{r_{i,j}}\right) \tag{14}$$

$r_{i,j}$ is the separations between conductors i and j; and $\sqrt{\frac{\mu}{\varepsilon}}$ is the intrinsic impedance.

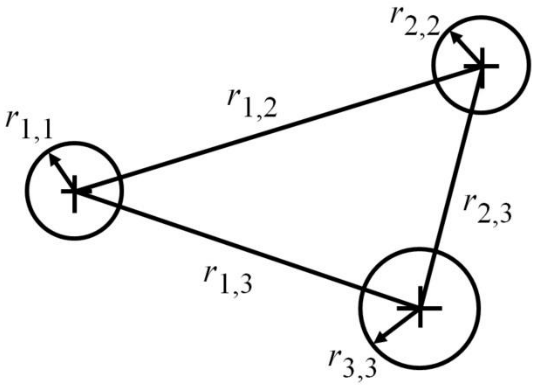

Figure 5 Cross section of conductors in a three-conductor assembly

3 Loop Equations

In practical situations, measurements using test equipment involve loops. Voltages Va_1 and Va_2 are measured between two adjacent terminals; and currents Ia_1 and Ia_2 flow out of one terminal and back via the other. Figure 6 defines the relationship between loop parameters and primitive parameters.

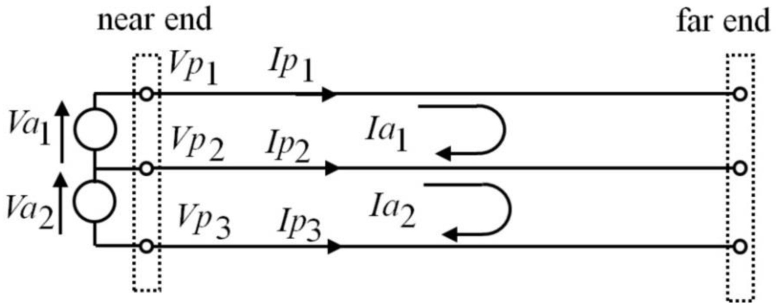

Figure 6 Relationship between loop parameters and primitive parameters

Relationships between primitive currents and loop currents are:

$$\begin{aligned} Ip_1 &= Ia_1 \\ Ip_2 &= Ia_2 - Ia_1 \\ Ip_3 &= -Ia_2 \end{aligned} \tag{15}$$

Relationships between loop voltages and primitive voltages are

$$\begin{aligned} Va_1 &= Vp_1 - Vp_2 \\ Va_2 &= Vp_2 - Vp_3 \end{aligned} \tag{16}$$

This leads to the Loop Equations

$$\begin{aligned} Va_1 &= Za_{1,1} \cdot Ia_1 + Za_{1,2} \cdot Ia_2 \\ Va_2 &= Za_{2,1} \cdot Ia_1 + Za_{2,2} \cdot Ia_2 \end{aligned} \tag{17}$$

Where

$$\begin{aligned} Za_{1,1} &= Zp_{1,1} - Zp_{1,2} - Zp_{2,1} + Zp_{2,2} \\ Za_{1,2} &= Zp_{1,2} - Zp_{1,3} - Zp_{2,2} + Zp_{2,3} \\ Za_{2,1} &= Zp_{2,1} - Zp_{2,2} - Zp_{3,1} + Zp_{3,2} \\ Za_{2,2} &= Zp_{2,2} - Zp_{2,3} - Zp_{3,2} + Zp_{3,3} \end{aligned} \tag{18}$$

4 Circuit Equations

Figure 7 illustrates a circuit model which can replicate the response of the Loop Equations. The Circuit Equations are:

$$\begin{aligned} Va_1 &= \left(Zc_1 + Zc_2\right) \cdot Ia_1 - Zc_2 \cdot Ia_2 \\ Va_2 &= -Zc_2 \cdot Ia_1 + \left(Zc_2 + Zc_3\right) \cdot Ia_2 \end{aligned} \tag{19}$$

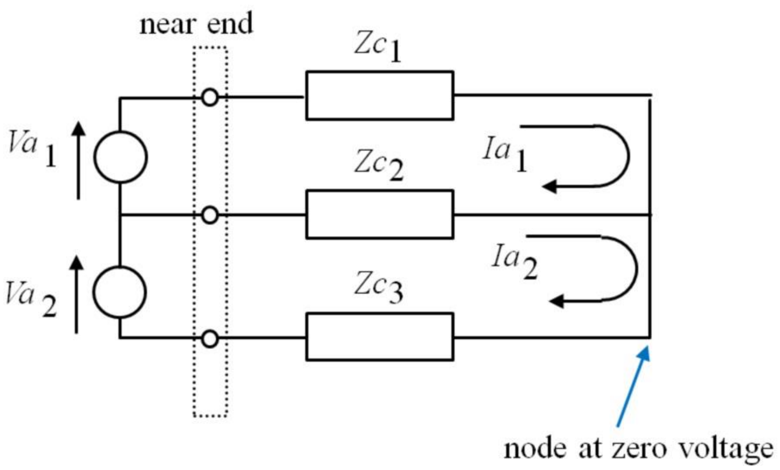

Figure 7 Circuit model which replicates the Loop Equations

Correlating the Z-parameters of equation (17) and (19) gives

$$\begin{aligned} Za_{1,1} &= Zc_1 + Zc_2 \\ Za_{1,2} &= -Zc_2 = Za_{2,1} \\ Za_{2,2} &= Zc_2 + Zc_3 \end{aligned} \tag{20}$$

The circuit parameters can now be defined in terms of the loop parameters

$$\begin{aligned} Zc_1 &= Za_{1,1} + Za_{1,2} \\ Zc_2 &= -Za_{1,2} \\ Zc_3 &= Za_{2,2} + Za_{1,2} \end{aligned} \tag{21}$$

Using (18) to replace the loop parameters with primitive parameters

$$\begin{aligned} Zc_1 &= Zp_{1,1} - Zp_{2,1} - Zp_{1,3} + Zp_{2,3} \\ Zc_2 &= Zp_{2,2} - Zp_{1,2} - Zp_{2,3} + Zp_{1,3} \\ Zc_3 &= Zp_{3,3} - Zp_{3,1} - Zp_{2,3} + Zp_{2,1} \end{aligned} \tag{22}$$

Values for the capacitors and the inductors can be derived from

$$\begin{aligned} Cc_k &= \frac{T}{Zc_k} \\ Lc_k &= T \cdot Zc_k \end{aligned} \tag{23}$$

Where the number k identifies the conductor and

$$T = \frac{l}{v} = l \cdot \sqrt{\mu \cdot \varepsilon} \tag{24}$$

T is the time taken for the front edge of a signal to propagate from one end of the line to the other and v is the velocity of propagation of the signal

This leads to:

$$\begin{aligned} Cc_1 &= \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\dfrac{r_{1,2} \cdot r_{1,3}}{r_{1,1} \cdot r_{2,3}}\right)} \\ Cc_2 &= \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\dfrac{r_{1,2} \cdot r_{2,3}}{r_{2,2} \cdot r_{1,3}}\right)} \\ Cc_3 &= \frac{2 \cdot \pi \cdot \varepsilon \cdot l}{\ln\left(\dfrac{r_{1,3} \cdot r_{2,3}}{r_{3,3} \cdot r_{1,2}}\right)} \end{aligned} \tag{25}$$

and

$$
\begin{aligned}
Lc_1 &= \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{r_{1,2} \cdot r_{1,3}}{r_{1,1} \cdot r_{2,3}}\right) \\
Lc_2 &= \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{r_{1,2} \cdot r_{2,3}}{r_{2,2} \cdot r_{1,3}}\right) \\
Lc_3 &= \frac{\mu \cdot l}{2 \cdot \pi} \cdot \ln\left(\frac{r_{1,3} \cdot r_{2,3}}{r_{3,3} \cdot r_{1,2}}\right)
\end{aligned}
\tag{26}
$$

Since the input impedance will be the same when the source voltage is applied to the far end, the circuit model must be symmetrical. This leads to Figure 8.

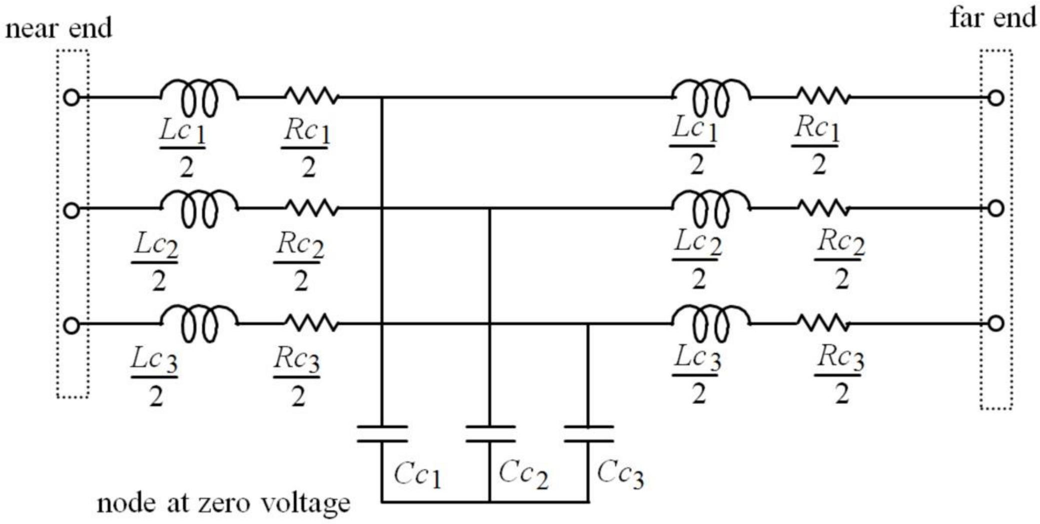

Figure 8 Circuit model of three-conductor assembly

This model allows all forms of intra-system coupling to be analysed. One of the conductors can simulate the effect of the conducting structure, plus the effect of all other conductors in the system. If the structure is used as a general-purpose return conductor, then the model can simulate the coupling between the culprit and victim loops. In this case, the structure acts as a common-impedance.

If a dedicated return conductor is used to carry the signal or power, then the model will simulate the coupling between the differential-mode loop and the common-mode loop. In this configuration, the structure acts as a shield.

5 Conclusion

It has been shown that the circuit model is a fictitious construct. When this is accepted as an irrefutable fact, it becomes possible to utilise the analytical tools of Circuit Theory to solve any and all EMI problems. Until this fact is accepted, we are destined to wander about in the dark.