Updating Circuit Theory

Transmission line model

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1 Introduction

Back in the time when Circuit Theory was in its infancy, the existence of electromagnetic interference (EMI) was deemed to be of little significance. Engineering judgement at that time was that it could safely be ignored. Since then, the development of valves, transistors, integrated circuits and the Internet of Things has created a situation where ignoring EMI is no longer an option. But during this time, the fundamentals of Circuit Theory have remained unchanged.

The time has come for that this theory to be updated, to provide electrical engineers with the analytical tools to deal with EMC requirements as a matter of routine.

Even though EMI is detectable in the behaviour of every electronic system, it is a topic that equipment designers are wary about discussing. During the development process for any new equipment, those aspects of design which relate to the EMC regulations tend to be outsourced to an external consultant. This approach is encouraged by Centres of Learning where EMI is characterised as an extremely complex topic, only understood by the gifted few. Full-field simulation is identified as the only dependable way to analyse the phenomenon.

Circuit Theory is a development and simplification of Electromagnetic Theory. But one of the simplifications is the concept of the equipotential ground. This is 'a conducting surface at which all points are at zero voltage'. The concept manifests itself as the ubiquitous earth symbol in many circuit diagrams. This has led inexorably to the concept of the single-point ground; 'a terminal on the conducting structure to which all voltages in the system are referred'.

Attempts to reconcile these concepts with the observed existence of interference have led to the publication of many books on EMC. However, these include erudite formulae gleaned from papers on Computational Electromagnetics, a topic which has as much meaning to the average designer as the hieroglyphics on a scroll of papyrus.

Such confusion can be avoided by going back to basics and reviewing the lessons learnt as a student. The relationship between Electromagnetic Theory and Circuit Theory is that the former defines all the fundamental equations and the latter provides the analytical tools necessary to solve those equations. So the task in hand is to identify the mechanisms involved in the propagation of EMI and to investigate how these mechanisms can be simulated using circuit models.

This particular article shows how a circuit model can be created to simulate the behaviour of a two-conductor cable over the entire range of frequencies covered by transmission line theory. A step-by-step derivation of the model is defined in a document which can be downloaded from http://www.designemc.info/24Transforms.pdf . The following description can be treated as guide to the path followed in the document, identifying the key relationships along the way.

2 Forward Propagation

Figure 1 illustrates a schematic diagram of a twin-conductor transmission line. It is assumed that the cross-section of the conductor assembly is constant along the entire length. The length of the line is *l*, there is a voltage source *Vn* at the near end, and this source delivers a current *In* to the line. The current delivered to the load at the far end of the line is *If* and the voltage between the terminals at the far end is *Vf*.

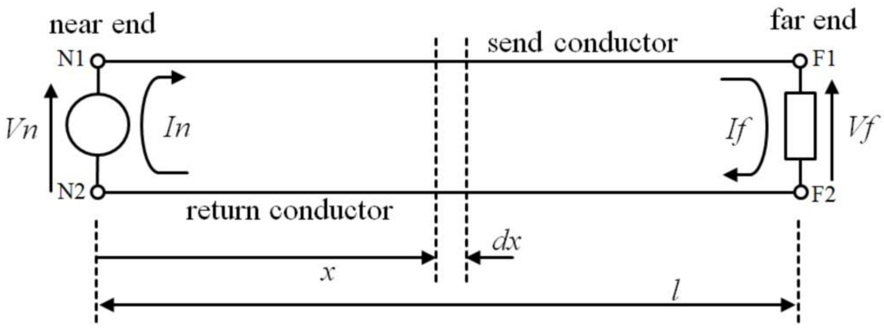

Figure 1 Schematic diagram of a twin-conductor transmission line

It is assumed that this line can be simulated as a number of LCR networks connected in series, and that one such segment is as shown by Figure 2. This represents the properties of the length *dx* at a distance *x* from the near end. Since each conductor posesses the properties of resistance and inductance, it is necessary to assign these properties to both.

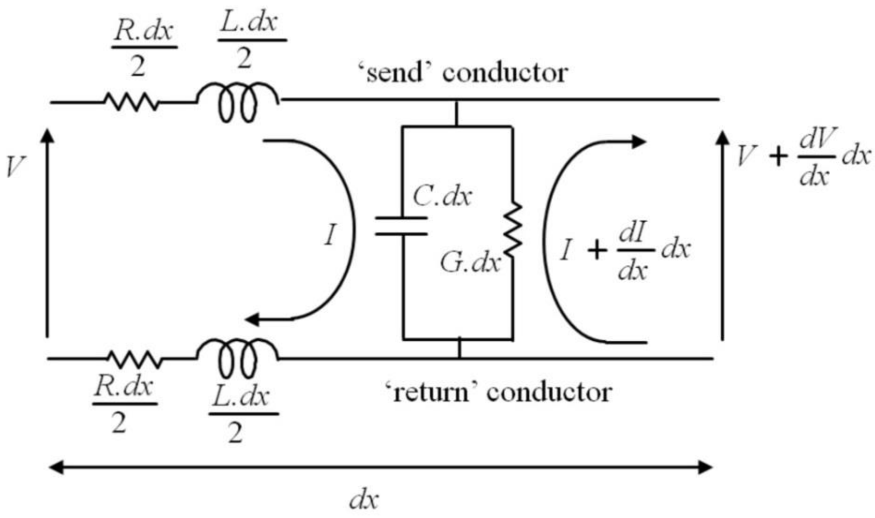

Figure 2 Voltages and currents in a single segment of the line

The parameters R,L, C, and G are defined as Ω/m, H/m, F/m and Seimens/m. Analysis of Figure 2 leads to:

$$\begin{aligned}(R + j\cdot\omega\cdot L)\cdot dx\cdot I &= V - \left(V + \frac{\delta V}{\delta x}\cdot dx\right)\\ (G + j\cdot\omega\cdot C)\cdot dx\cdot V &= I - \left(I + \frac{\delta I}{\delta x}\cdot dx\right)\end{aligned} \tag{1}$$

It is clear from Figure 2 that the each current flows in a loop. That is, current flows in both directions through the insulation. The send conductor can be visualised as a transmitting antenna. Charges are deposited on the return conductor. This creates current flow in the return conductor. It behaves as a receiving antenna. Current flowing in the opposite direction to that in the send conductor creates a magnetic field. This enhances the forward flow of current in the send conductor. The field so created between the conductors acts to steer the electromagnetic energy along the path defined by the routing of the cable. This means that the inductance of the return conductor plays a crucial role in the behaviour of any electronic system.

Analysis of the differential equations leads to one solution:

$$\begin{aligned}V &= Vn\cdot e^{-\gamma\cdot x}\\ I &= In\cdot e^{-\gamma\cdot x}\end{aligned} \tag{2}$$

where $$\gamma = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)} \tag{3}$$

More mathematical manipulation leads to:

$$I = \frac{V}{Zo} \tag{4}$$

where $$Zo = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}} \tag{5}$$

γ is named the propagation constant and Zo is defined as the characteristic impedance.

Another significant relationship which emerges is:

$$\frac{\delta I}{\delta x} = \frac{1}{Zo}\cdot\frac{\delta V}{\delta x} \tag{6}$$

This means that the rate of change of current is proportional to the rate of change of voltage. Since the current is also proportional to the voltage, there can be no phase difference between the current and voltage waveforms during forward propagation. Figure 3 illustrates this. A further deduction is that the charge propagates forward at the same velocity as the voltage and current.

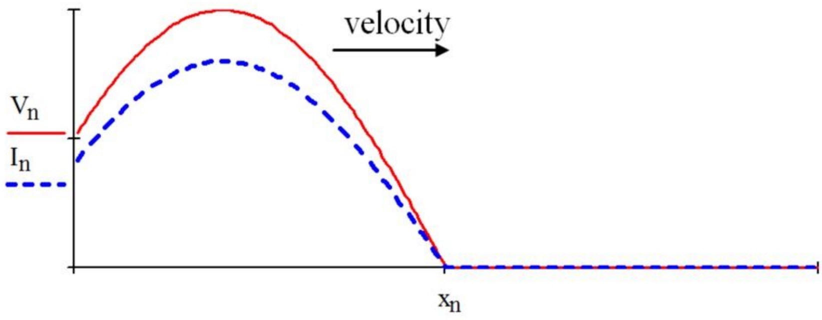

Figure 3 Relationship between voltage and current waveforms

3 Two-way propagation

When the signal reaches the termination at the far end, some of it is absorbed by the load, and some if it is reflected. This reflected signal flows from the far end to the near end. Energy is flowing in both directions along the line. So the total voltage at any point is given by the equation:

$$V = H \cdot e^{\gamma \cdot x} + K \cdot e^{-\gamma \cdot x} \tag{7}$$

It is also possible to define the relationship as:

$$V = A \cdot \cosh(\gamma \cdot x) + B \cdot \sinh(\gamma \cdot x) \tag{8}$$

because:

$$\begin{aligned} V &= A \cdot \left(\frac{e^{\gamma \cdot x} + e^{-\gamma \cdot x}}{2} \right) + B \cdot \left(\frac{e^{\gamma \cdot x} - e^{-\gamma \cdot x}}{2} \right) \\ &= \frac{A+B}{2} \cdot e^{\gamma \cdot x} + \frac{A-B}{2} \cdot e^{-\gamma \cdot x} \\ &= H \cdot e^{\gamma \cdot x} + K \cdot e^{-\gamma \cdot x} \end{aligned}$$

The next task is to assign values to the parameters *A* and *B*. From an engineering point of view, the only voltages and currents of any interest are those which exist at the interfaces at the near and far ends. If these are as defined by Figure 1, then the mathematics leads to:

$$\begin{aligned} Vn &= Vf \cdot \cosh(\gamma \cdot l) + Zo \cdot If \cdot \sinh(\gamma \cdot l) \\ In &= \frac{Vf}{Zo} \cdot \sinh(\gamma \cdot l) + If \cdot \cosh(\gamma \cdot l) \end{aligned} \tag{9}$$

Where *l* is the length of the cable between the near end and the far end. The derivation of the of hybrid equations is usually as far as books on Electromagnetic Theory go. At this point they go on to derive the reflection coefficients, the quarter-wave transformer and to describe the Smith Chart.

The significant feature of the hybrid equations is that they invoke the relationships of Electromagnetic Theory to define the parameters for the complete line.

4 The Transformation Equations

Such a configuration can also be defined by the circuit model of Figure 4.

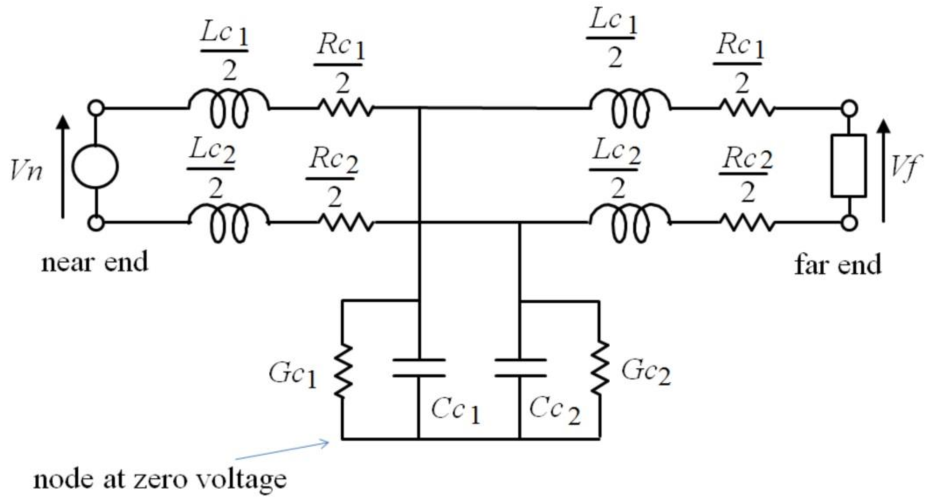

Figure 4 Lumped parameter model

Invoking the concept of equivalent circuits, this can be simplified to that shown on Figure 5a, where:

$$\begin{aligned} Rd &= Rc_1 + Rc_2 \\ Ld &= Lc_1 + Lc_2 \\ \frac{1}{Cd} &= \frac{1}{Cc_1} + \frac{1}{Cc_2} \\ \frac{1}{Gd} &= \frac{1}{Gc_1} + \frac{1}{Gc_2} \end{aligned} \tag{10}$$

<u>This simplification does not mean that the line at the bottom of Figure 2 actually represents an equipotential conductor</u>. The relationship between the per-unit-length parameters and the lumped parameters is:

$$\begin{aligned} Rd &= R \cdot l \\ Ld &= L \cdot l \\ Cd &= C \cdot l \\ Gd &= G \cdot l \end{aligned} \tag{11}$$

If the parameter θ is defined as

$$\theta = \lambda \cdot l \tag{12}$$

Then the characteristic impedance and the propagation constant can be related to parameters which can be measured using electronic test equipment. There is now no need to invoke the use of per-unit-length parameters during the analysis of transmission lines.

$$\begin{aligned} \theta &= \sqrt{(Rd + j\cdot\omega\cdot Ld)\cdot(Gd + j\cdot\omega\cdot Cd)} \\ Zo &= \sqrt{\frac{Rd + j\cdot\omega\cdot Ld}{Gd + j\cdot\omega\cdot Cd}} \end{aligned} \tag{13}$$

Mesh analysis of Figure 5a leads to the loop equations:

$$\begin{aligned} Vn &= (Zh + Zv)\cdot In - Zv\cdot If \\ Vf &= Zv\cdot In - (Zh + Zv)\cdot If \end{aligned} \tag{14}$$

Manipulating these equations into the same form as the hybrid equations leads to:

$$\begin{aligned} Vn &= \left(1 + \frac{Zh}{Zv}\right)\cdot Vf + Zh\cdot\left(2 + \frac{Zh}{Zv}\right)\cdot If \\ In &= \frac{Vf}{Zv} + \left(1 + \frac{Zh}{Zv}\right)\cdot If \end{aligned} \tag{15}$$

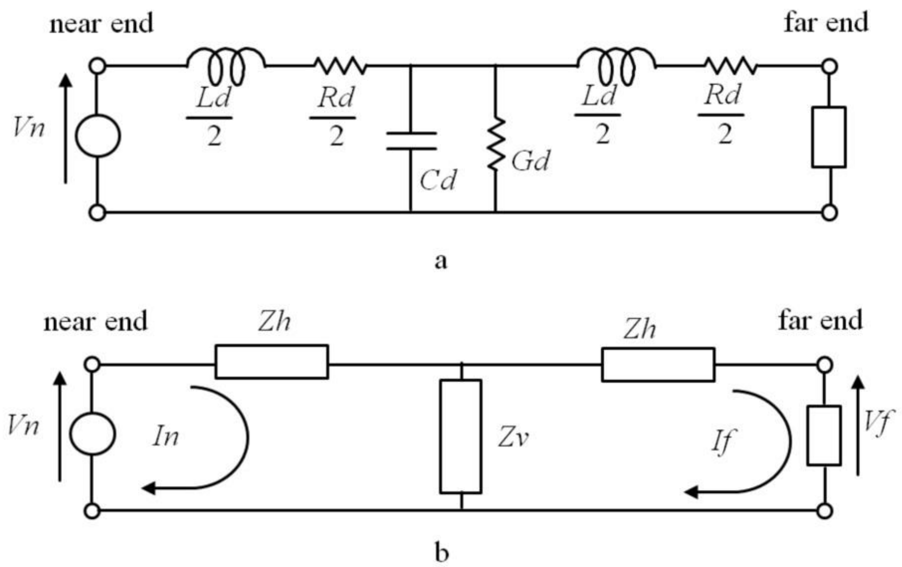

Figure 5 Single-T network models of a signal link
a Lumped parameter model
b Distributed-parameter model

There are now two pairs of hybrid equations; one derived from Electromagnetic Theory and one derived from Circuit Theory. Correlating the parameters common to both sets leads to

$$\begin{aligned} Zh &= Zo \cdot \tanh\left(\frac{\theta}{2}\right) \\ Zv &= Zo \cdot \text{cosech } \theta \end{aligned} \tag{16}$$

There is now a clear correlation between Figures 5a and 5b.

$$\begin{aligned} \frac{\left(Rd + j \cdot \omega \cdot Ld\right)}{2} &\rightarrow Zo \cdot \tanh\left(\frac{\theta}{2}\right) \\ \frac{1}{Gd + j \cdot \omega \cdot Cd} &\rightarrow Zo \cdot \text{cosech}\, \theta \end{aligned} \tag{17}$$

This means that Figure 5a can be used to define the parameters which characterise the transmission line and Figure 5b can be used to simulate the response.

5 Propagation velocity.

Since γ is a complex number, it can be defined as

$$\gamma = \alpha + j \cdot \beta \tag{18}$$

where α represents the loss constant and β represents the phase constant.

All the voltage and currents so far defined are phasors. Hence

$$Vt = Vpeak \cdot e^{j \cdot \omega \cdot t} \cdot e^{-(\alpha + j \cdot \beta) \cdot x} \tag{19}$$

Where *Vpeak* is the peak value of the waveform and *Vt* is a function of time and distance. Re-arranging this:

$$Vt = Vpeak \cdot e^{-\alpha \cdot x} \cdot e^{j \cdot (\omega \cdot t - \beta \cdot x)}$$

if

$$\omega \cdot t - \beta \cdot x = A \tag{20}$$

where *A* is a constant angle, then

$$x = \frac{\omega \cdot t}{\beta} - \frac{A}{\beta} \tag{21}$$

and

$$\frac{dx}{dt} = \frac{\omega}{\beta} = v \tag{22}$$

where *v* is the velocity of propagation.

With a lossless line, $Rd = 0$, $Gd = 0$ and $\alpha = 0$. Also,

$$\gamma = \alpha + j \cdot \beta = \frac{1}{l} \cdot \sqrt{j \cdot \omega \cdot Ld \cdot j \cdot \omega \cdot Ld} = j \cdot \frac{\omega}{l} \cdot \sqrt{Ld \cdot Cd}$$

that is

$$\beta = \frac{\omega}{l} \cdot \sqrt{Ld \cdot Cd} \quad (23)$$

$$l = \frac{\omega}{\beta} \cdot \sqrt{Ld \cdot Cd} = v \cdot \sqrt{Ld \cdot Cd}$$

giving

$$\sqrt{Ld \cdot Cd} = \frac{l}{v} = T \quad (24)$$

where T is the time taken for a pulse to propagate from one end of the line to the other.

6 Conclusion

A circuit model has been derived to simulate the behaviour of a two-conductor cable over the entire frequency range covered by transmission line theory.

It has been shown that the return conductor possesses the properties of inductance and capacitance and that it plays an essential role in propagating the electromagnetic energy along the path defined by the routing of the cable