New T model incorporating conductor and substrate parasitic losses for on-chip transformers

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The demand for low-power, low-cost and high-integration wireless communication systems is driving the development of on-chip passive devices for radio frequency integrated circuits (RFICs).

Among the passive circuit designs, on-chip spiral transformers are particularly important and widely used in RFICs, including mixers, low-noise amplifiers, power amplifiers and oscillators. This calls for accurate lumped-element models for on-chip transformers, suitable for circuit simulation and design optimisation. Modelling the transformer and obtaining its parameters are always of great concern for engineers and circuit designers.

Several compact transformer models and circuit parameter-determining approaches have been proposed over the years, including a double π model that has 42 components and expressions for spiral transformers; an automatic parameter determination and scaleable modelling method; and a generalised four-port transformer model with an ideal transformer at its core.

Commonly-used transformer model specifications don't include high-order effects with increasing frequency, which can result in underestimated power consumption of the circuit model when compared to the actual transformer's performance.

Certain physics-based modelling methods for calculating the transformer model parameters from geometry, and the necessary semiconductor process have also been developed. However, these process parameters are not always well defined for modelling purposes. In addition, some expressions are merely empirical formulas that depend on a specific fabrication process. Thus, some of the models' parameters are totally or partially determined by fitting and optimisation procedures.

We propose a new T-lumped-element circuit model for circle-shaped planar transformers. This model accurately captures conductor losses in the transformer windings as well as substrate parasitic losses over a broad frequency range. The values of the T-lumped elements of the circuit model are determined from two-port S-parameter data obtained from measurements.

This model accurately captures conductor losses in the transformer windings

Model Development

Figure 1 shows the proposed corresponding physics-based equivalent circuit model for on-chip spiral transformers. Here, R_{i0} (i = p, s) is introduced to account for the spiral coil's resistive loss, and C_{i0} (i = p, s) is the coupling capacitance between adjacent metal tracks and the overlap capacitance between the spiral and underpass metal lines. Direct current is uniformly distributed inside the conductor, characterised by R_i and L_i (i = p, s). As the frequency goes up, skin and proximity effects will push the

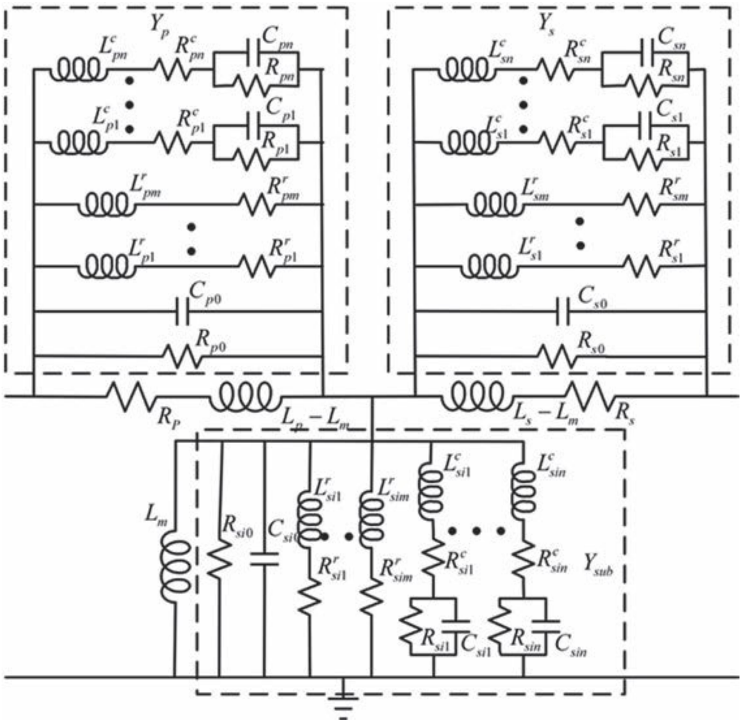

Figure 1: New compact model for transformers

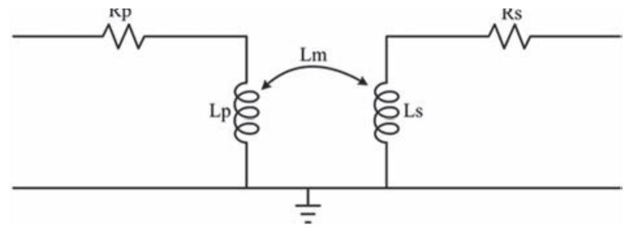

Figure 2: Physical model for low-frequency bands

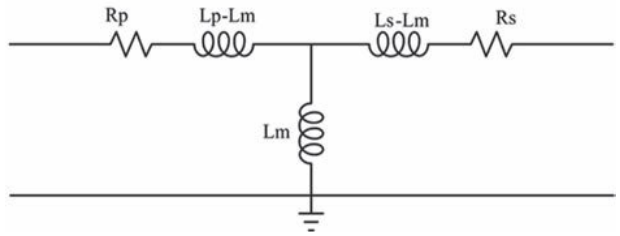

Figure 3: Decoupling conversion

alternating current to the surface of the metal, requiring an additional L_R and L_{RC} ladder in parallel with R_i and L_i to capture the different current densities in the conductor.

The conventional substrate parasitic can be modelled by C_{ox}, C_{sub} and R_{sub}, with C_{ox} representing the metal-oxide capacitance and C_{sub} and R_{sub} the substrate capacitance and resistance. But, for substrate high-order effects such as the eddy effect, characterising the capacitive and resistive couplings alone is not enough.

A new block shown as Y_{sub} in Figure 1 was introduced to model substrate parasitic losses, where R_{si0} is the substrate resistance, C_{si0} the capacitance of the metal-oxide and substrate, and an LR and LRC series branch accounts for the model's high-order effects. This block significantly improves the accuracy of the model. More branches will describe the high-order effects more accurately.

The magnetic couplings between the two coils are represented by L_m.

For convenience, the portions including R_{i0}, C_{i0}, L_R and L_{RC} in the series branch of the equivalent circuit will be denoted by Y_i (i = p, s), and the sections including C_{si0}, R_{si0}, L_R and L_{RC} in the shunt branch by Y_{sub}, as indicated by the dashed lines in Figure 1.

Parameter Determination

In low-frequency bands, the physical transformer model can be described as in Figure 2. Here R_i and L_i (i = p, s) denote the DC resistance and inductance of the corresponding coil, respectively; L_m is introduced to account for the mutual inductance between the primary and secondary coils. We can easily convert this π topology to the T topology and determine R_i, L_i (i = p, s) and L_m with:

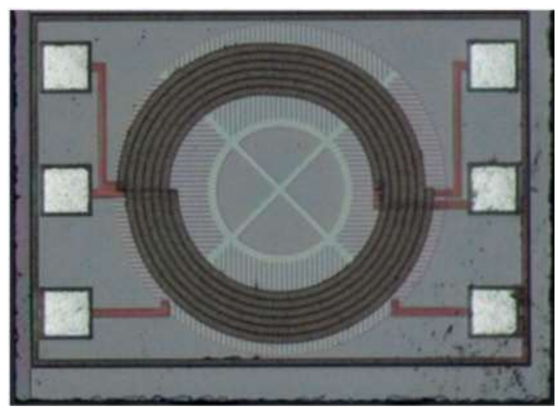

Figure 4: Top view of the fabricated on-chip transformer

$$L_m = \left.\frac{im\left[(Z_{12}+Z_{21})/2\right]}{w}\right|_{w\to 0} \tag{1}$$

$$R_i = re(Z_{ii}-Z_{12})\big|_{w\to 0}\,(i=p,s) \tag{2}$$

$$L_i = L_m + \left.\frac{im(Z_{ii}-Z_{12})}{w}\right|_{w\to 0}\quad(i=p,s) \tag{3}$$

The circuit parameter determination of the proposed model is mainly for Y_p, Y_s and Y_{sub}. When the elements L_p, R_p, L_s, R_s and L_m are determined, the Y_i (i = p, s) and Y_{sub} can be calculated with:

$$Y_i = \frac{1}{Z_{ii}-Z_{12}} - \frac{1}{R_i + jwL_i}(i=p,s) \tag{4}$$

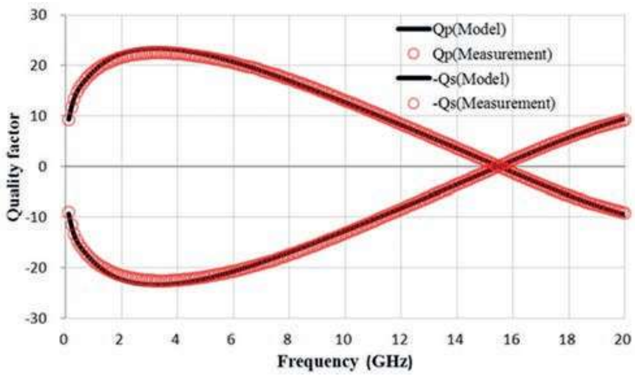

Figure 5: Quality factor comparisons

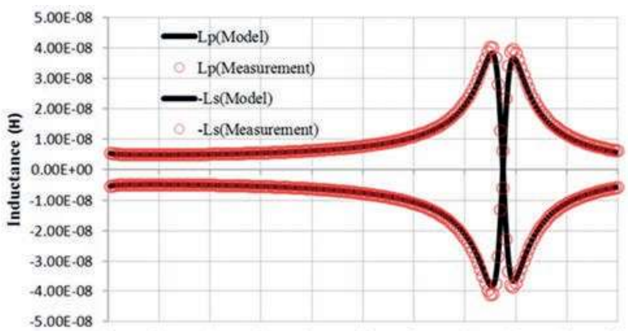

Figure 6: Inductance comparisons

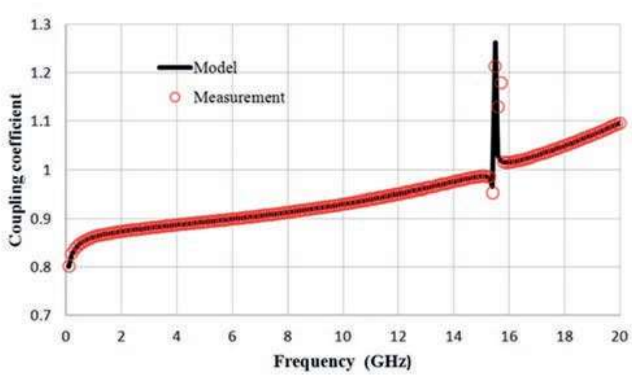

Figure 7: Coupling coefficient comparisons

$$Y_{sub} = \frac{2}{Z_{12} + Z_{21}} - \frac{1}{jwL_m} \tag{5}$$

Expanding the admittance of Y_i and Y_{sub} with the pole-residue formulation will implement the parameter determination process simply:

$$\begin{aligned} Y(s) &= d + se + \sum_{i=1}^{m} \frac{r_i^r}{s - p_i^r} + \sum_{i=1}^{n} \frac{r_i^c}{s - p_i^c} + \frac{\tilde{r}_i^c}{s - \tilde{p}_i^c} \\ &= d + se + \sum_{i=1}^{m} \frac{r_i}{s - p_i} + \sum_{i=1}^{n} \frac{\lambda_i s + \gamma_i}{s^2 + \alpha_i s + \beta_i} \end{aligned} \tag{6}$$

In Equation 6, d, e, r_i, p_i, α_i, β_i, γ_i and λ_i are real, with the rest complex. To synthesise a lumped equivalent circuit by Y(s), Y_j (j = p, s, si) is expressed by R, L, C as:

$$\begin{aligned} Y_j(s) &= \frac{1}{R_{j0}} + sC_{j0} + \sum_{i=1}^{m} \frac{1/L_{ji}^r}{s + R_{ji}^r / L_{ji}^r} \\ &+ \sum_{i=1}^{n} \frac{(1 + L_{ji}^c)s + 1/L_{ji}^c C_{ji} R_{ji}}{s^2 + (R_{ji}^c / L_{ji}^c + 1/C_{ji}R_{ji})s + (1 + R_{ji}^c / R_{ji})/(L_{ji}^c C_{ji})} \end{aligned} \tag{7}$$

The superscripts r and c stand for real and complex poles. The order of real and complex poles are m and n. Compared with the aforementioned Y(s), the R, L, C components of the circuit in Figure 1 can be determined with:

$$\left.\begin{aligned} R_{j0} &= 1/d \\ C_{j0} &= e \end{aligned}\right\} (j = p, s, si) \tag{8}$$

$$\left.\begin{aligned} L_{ji}^r &= 1/r_i^r \\ R_{ji}^r &= -p_i^r / r_i^r \end{aligned}\right\} (j = p, s, si; i = 1 \cdots m) \tag{9}$$

$$\left.\begin{aligned} L_{ji}^c &= 1/\lambda_i \\ R_{ji}^c &= L_{ji}^c(\alpha_i - L_{ji}\gamma_i) \\ R_{ji} &= \beta_i / \gamma_i - R_{ji}^c \\ C_{ji} &= 1/L_{ji}^c R_{ji} \gamma_i \end{aligned}\right\} (j = p, s, si, i = 1 \cdots n) \tag{10}$$

Therefore, all the elements in Y_p, Y_s and Y_{sub} are identified. This parameter-determining method has been implemented in the MATLAB environment using the VF (vector fitting) method.

Parameter	Value	Parameter	Value
R_p, R_s/Ω	62	L_m/nH	4.26
L_p, L_s/nH	2.6	R_{si0}/kΩ	1.28
C_{p0}, C_{s0}/fF	3.56	C_{si0}/fF	27.8
R_{p0}, R_{s0}/Ω	667	L_{si1}^r/nH	260
L_{p1}^r, L_{s1}^r/nH	2.37	R_{si1}^r/Ω	203
R_{p1}^r, R_{s1}^r/Ω	11.5	L_{si2}^r/nH	212
L_{p2}^r, L_{s2}^r/nH	2.12	R_{si2}^r/kΩ	1.15
R_{p2}^r, R_{s2}^r/Ω	2.15	L_{si3}^r/nH	2.4
L_{p3}^r, L_{s3}^r/nH	1.55	R_{si3}^r/kΩ	1.04
R_{p3}^r, R_{s3}^r/Ω	0.16	L_{si1}^c/uH	9.53
L_{p1}^c, L_{s1}^c/nH	2.61	R_{si1}^c/kΩ	0.29
R_{p1}^c, R_{s1}^c/Ω	4.93	R_{si1}/kΩ	2.27e4
R_{p1}, R_{s1}/kΩ	299	C_{si1}/fF	1.47
C_{p1}, C_{s1}/fF	1.77		

Table 1: The determined model parameters

Model Verification

To verify the model's performance, an on-chip interleaved transformer with PGS (patterned ground shield) has been fabricated in a 0.5µm L50G CMOS process; the line width and spacing are 10 and 2µm respectively, and the transformer's inner diameter is 130µm, as shown in Figure 4. The measured S-parameter is extracted with a two-step (open and short) procedure to remove the undesired pad parasitics.

We used Z parameters converted from the measured S-parameters to determine the self-inductance (L_p and L_s), quality factors (Q_p and Q_s) and coupling coefficient (k) as functions of frequency. These are calculated using the following expressions:

$$L_p = \frac{im(Z_{11})}{2\pi f} \qquad (11)$$

$$L_s = \frac{im(Z_{22})}{2\pi f} \qquad (12)$$

$$Q_P = \frac{im(Z_{11})}{re(Z_{11})} \qquad (13)$$

$$Q_S = \frac{im(Z_{22})}{re(Z_{22})} \qquad (14)$$

$$k = \sqrt{\frac{im(Z_{12})im(Z_{21})}{im(Z_{11})im(Z_{22})}} \qquad (15)$$

To verify the procedure's efficiency and accuracy, we have compared the modelled and measured L_p, L_s, Q_p, Q_s and k. It only takes four parallel branches (three branches for real poles and one for complex poles) to represent the high-order effects. The model parameters generated by the direct determination procedure are listed in Table 1.

For clarity in Figures 5 and 6, we compared the curves of $-L_s$ and $-Q_s$ instead of L_s and Q_s. RMS deviations of the simulated Q_p and Q_s in Figure 5 are below 2.59%, the simulated L_p and L_s in Figure 6 are below 6.89%, and the simulated k value in Figure 7 is 7.61%.

The frequency range chosen for the RMS calculation is between 0.1GHz and 20GHz. The relative deviation is below 1.13% for the DC inductance L, and below 3.47% for the peak value of Q. As demonstrated in these three figures and deviations, an excellent agreement has been found over a broad frequency range; hence, the model in Figure 1 is accurate enough to model the transformer, with the parameter-determining procedure showing high efficiency and accuracy without any optimisation.

The equivalent circuit model is compatible with common circuit simulators such as SPICE, for transient and frequency domain simulation. EW