

# New T model incorporating conductor and substrate parasitic losses for on-chip transformers

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**T**he demand for low-power, low-cost and high-integration wireless communication systems is driving the development of on-chip passive devices for radio frequency integrated circuits (RFICs).

Among the passive circuit designs, on-chip spiral transformers are particularly important and widely used in RFICs, including mixers, low-noise amplifiers, power amplifiers and oscillators. This calls for accurate lumped-element models for on-chip transformers, suitable for circuit simulation and design optimisation. Modelling the transformer and obtaining its parameters are always of great concern for engineers and circuit designers.

Several compact transformer models and circuit parameter-determining approaches have been proposed over the years, including a double  $\pi$  model that has 42 components and expressions for spiral transformers; an automatic parameter determination and scaleable modelling method; and a generalised four-port transformer model with an ideal transformer at its core.

Commonly-used transformer model specifications don't include high-order effects with increasing frequency, which can result in underestimated power consumption of the circuit model when compared to the actual transformer's performance.

Certain physics-based modelling methods for calculating the transformer model parameters from geometry, and the necessary semiconductor process have also been developed. However, these process parameters are not always well defined for modelling purposes. In addition, some expressions are merely empirical

formulas that depend on a specific fabrication process. Thus, some of the models' parameters are totally or partially determined by fitting and optimisation procedures.

We propose a new T-lumped-element circuit model for circle-shaped planar transformers. This model accurately captures conductor losses in the transformer windings as well as substrate parasitic losses over a broad frequency range. The values of the T-lumped elements of the circuit model are determined from two-port S-parameter data obtained from measurements.

## This model accurately captures conductor losses in the transformer windings

### Model Development

Figure 1 shows the proposed corresponding physics-based equivalent circuit model for on-chip spiral transformers. Here,  $R_{i0}$  ( $i = p, s$ ) is introduced to account for the spiral coil's resistive loss, and  $C_{i0}$  ( $i = p, s$ ) is the coupling capacitance between adjacent metal tracks and the overlap capacitance between the spiral and underpass metal lines. Direct current is uniformly distributed inside the conductor, characterised by  $R_i$  and  $L_i$  ( $i = p, s$ ). As the frequency goes up, skin and proximity effects will push the

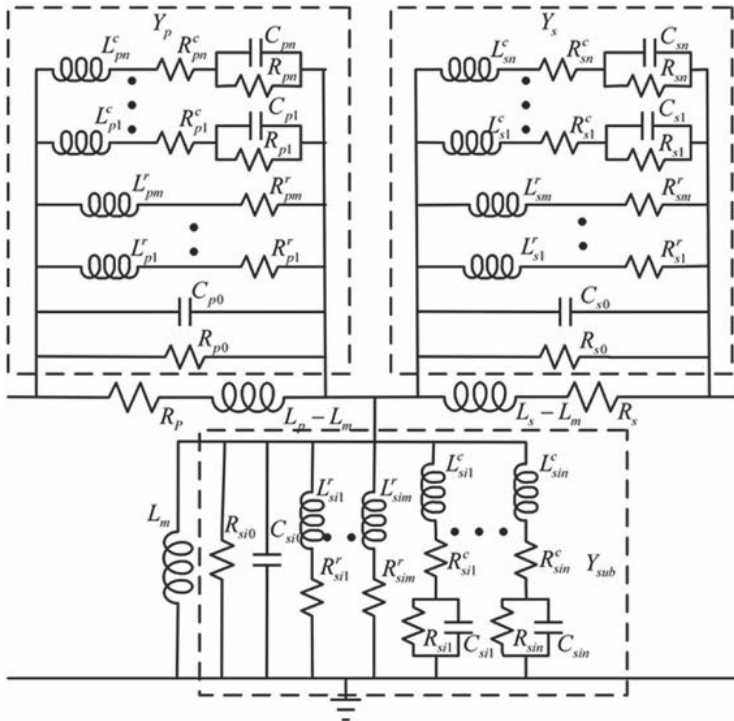


Figure 1: New compact model for transformers

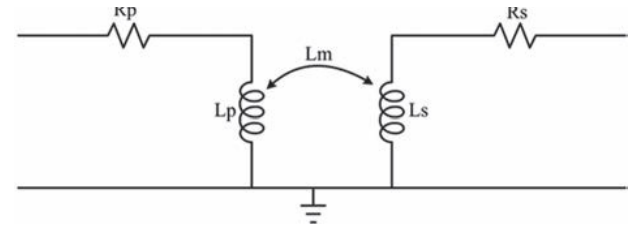


Figure 2: Physical model for low-frequency bands

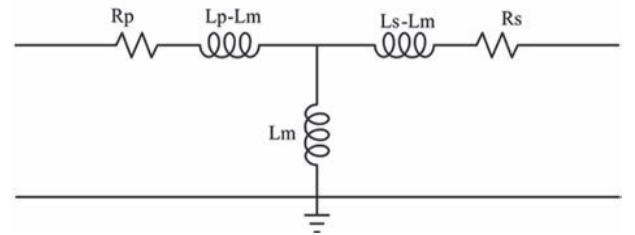


Figure 3: Decoupling conversion

alternating current to the surface of the metal, requiring an additional  $L_R$  and  $L_{RC}$  ladder in parallel with  $R_i$  and  $L_i$  to capture the different current densities in the conductor.

The conventional substrate parasitic can be modelled by  $C_{ox}$ ,  $C_{sub}$  and  $R_{sub}$ , with  $C_{ox}$  representing the metal-oxide capacitance and  $C_{sub}$  and  $R_{sub}$  the substrate capacitance and resistance. But, for substrate high-order effects such as the eddy effect, characterising the capacitive and resistive couplings alone is not enough.

A new block shown as  $Y_{sub}$  in Figure 1 was introduced to model substrate parasitic losses, where  $R_{si0}$  is the substrate resistance,  $C_{si0}$  the capacitance of the metal-oxide and substrate, and an LR and LRC series branch accounts for the model's high-order effects. This block significantly improves the accuracy of the model. More branches will describe the high-order effects more accurately.

The magnetic couplings between the two coils are represented by  $L_m$ .

For convenience, the portions including  $R_{i0}$ ,  $C_{i0}$ ,  $L_R$  and  $L_{RC}$  in the series branch of the equivalent circuit will be denoted by  $Y_i$  ( $i = p, s$ ), and the sections including  $C_{si0}$ ,  $R_{si0}$ ,  $L_R$  and  $L_{RC}$  in the shunt branch by  $Y_{sub}$ , as indicated by the dashed lines in Figure 1.

### Parameter Determination

In low-frequency bands, the physical transformer model can be described as in Figure 2. Here  $R_i$  and  $L_i$  ( $i = p, s$ ) denote the DC resistance and inductance of the corresponding coil, respectively;  $L_m$  is introduced to account for the mutual inductance between the primary and secondary coils. We can easily convert this  $\pi$  topology to the T topology and determine  $R_i$ ,  $L_i$  ( $i = p, s$ ) and  $L_m$  with:

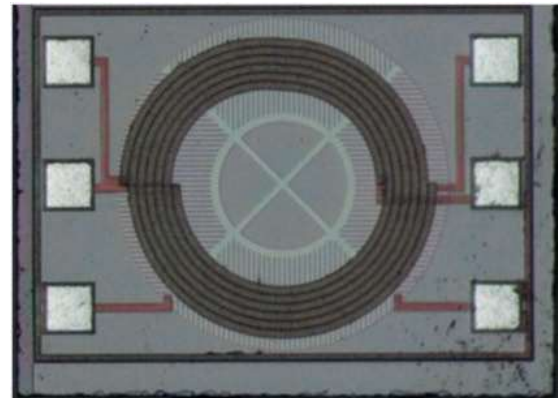


Figure 4: Top view of the fabricated on-chip transformer

$$L_m = \frac{im[(Z_{12} + Z_{21})/2]}{w} \Big|_{w \rightarrow 0} \quad (1)$$

$$R_i = re(Z_{ii} - Z_{12}) \Big|_{w \rightarrow 0} \quad (i = p, s) \quad (2)$$

$$L_i = L_m + \frac{im(Z_{ii} - Z_{12})}{w} \Big|_{w \rightarrow 0} \quad (i = p, s) \quad (3)$$

The circuit parameter determination of the proposed model is mainly for  $Y_p$ ,  $Y_s$  and  $Y_{sub}$ . When the elements  $L_p$ ,  $R_p$ ,  $L_s$ ,  $R_s$  and  $L_m$  are determined, the  $Y_i$  ( $i = p, s$ ) and  $Y_{sub}$  can be calculated with:

$$Y_i = \frac{1}{Z_{ii} - Z_{12}} - \frac{1}{R_i + jwL_i} \quad (i = p, s) \quad (4)$$

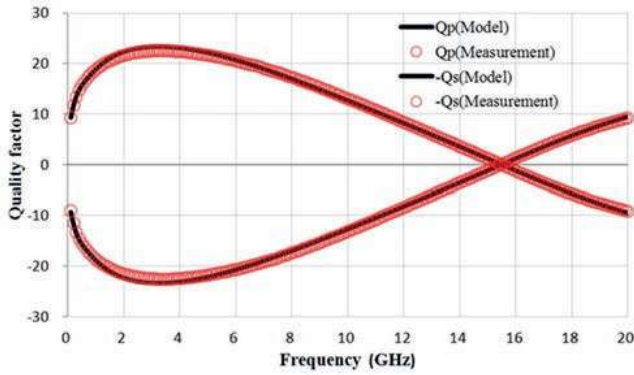


Figure 5: Quality factor comparisons

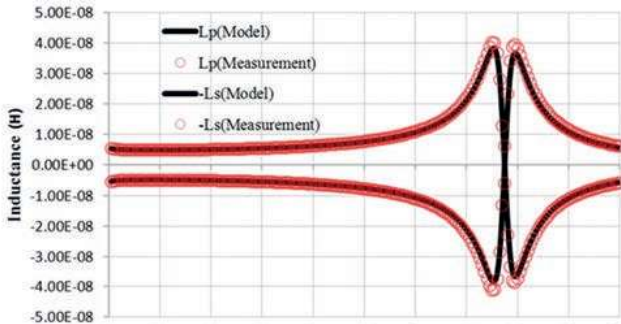


Figure 6: Inductance comparisons

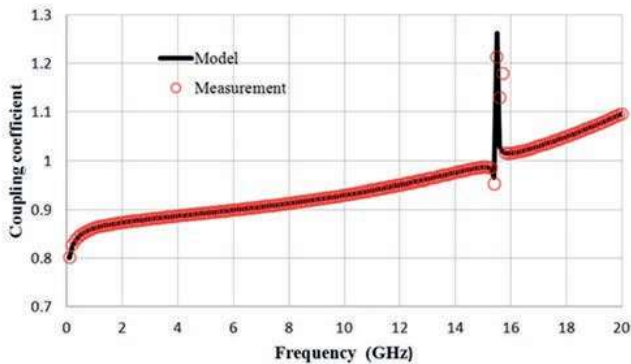


Figure 7: Coupling coefficient comparisons

$$Y_{sub} = \frac{2}{Z_{12} + Z_{21}} - \frac{1}{j\omega L_m} \quad (5)$$

Expanding the admittance of  $Y_1$  and  $Y_{sub}$  with the pole-residue formulation will implement the parameter determination process simply:

$$Y(s) = d + se + \sum_{i=1}^m \frac{r_i^r}{s - p_i^r} + \sum_{i=1}^n \frac{r_i^c}{s - p_i^c} + \frac{\tilde{r}_i^c}{s - \tilde{p}_i^c} \quad (6)$$

$$= d + se + \sum_{i=1}^m \frac{r_i}{s - p_i} + \sum_{i=1}^n \frac{\lambda_i s + \gamma_i}{s^2 + \alpha_i s + \beta_i}$$

In Equation 6,  $d, e, r_i, p_i, \alpha_i, \beta_i, \gamma_i$  and  $\lambda_i$  are real, with the rest complex. To synthesise a lumped equivalent circuit by  $Y(s)$ ,  $Y_j$  ( $j = p, s, si$ ) is expressed by  $R, L, C$  as:

$$Y_j(s) = \frac{1}{R_{j0}} + sC_{j0} + \sum_{i=1}^m \frac{1/L_{ji}^r}{s + R_{ji}^r/L_{ji}^r} \quad (7)$$

$$+ \sum_{i=1}^n \frac{(1 + L_{ji}^c)s + 1/L_{ji}^c C_{ji} R_{ji}}{s^2 + (R_{ji}^c/L_{ji}^c + 1/C_{ji} R_{ji})s + (1 + R_{ji}^c/R_{ji})/(L_{ji}^c C_{ji})}$$

The superscripts  $r$  and  $c$  stand for real and complex poles. The order of real and complex poles are  $m$  and  $n$ . Compared with the aforementioned  $Y(s)$ , the  $R, L, C$  components of the circuit in Figure 1 can be determined with:

$$\left. \begin{aligned} R_{j0} &= 1/d \\ C_{j0} &= e \end{aligned} \right\} (j = p, s, si) \quad (8)$$

$$\left. \begin{aligned} L_{ji}^r &= 1/r_i^r \\ R_{ji}^r &= -p_i^r/r_i^r \end{aligned} \right\} (j = p, s, si; i = 1 \dots m) \quad (9)$$

$$\left. \begin{aligned} L_{ji}^c &= 1/\lambda_i \\ R_{ji}^c &= L_{ji}^c(\alpha_i - L_{ji}\gamma_i) \\ R_{ji} &= \beta_i/\gamma_i - R_{ji}^c \\ C_{ji} &= 1/L_{ji}^c R_{ji} \gamma_i \end{aligned} \right\} (j = p, s, si; i = 1 \dots n) \quad (10)$$

Therefore, all the elements in  $Y_p, Y_s$  and  $Y_{sub}$  are identified. This parameter-determining method has been implemented in the MATLAB environment using the VF (vector fitting) method.

Parameter	Value	Parameter	Value
$R_p, R_s/\Omega$	62	$L_m/nH$	4.26
$L_p, L_s/nH$	2.6	$R_{si0}/k\Omega$	1.28
$C_{p0}, C_{s0}/fF$	3.56	$C_{si0}/fF$	27.8
$R_{p0}, R_{s0}/\Omega$	667	$L_{si1}/nH$	260
$L_{p1}, L_{s1}/nH$	2.37	$R_{si1}^r/\Omega$	203
$R_{p1}, R_{s1}/\Omega$	11.5	$L_{si2}/nH$	212
$L_{p2}, L_{s2}/nH$	2.12	$R_{si2}^r/k\Omega$	1.15
$R_{p2}, R_{s2}/\Omega$	2.15	$L_{si3}/nH$	2.4
$L_{p3}, L_{s3}/nH$	1.55	$R_{si3}^r/k\Omega$	1.04
$R_{p3}, R_{s3}/\Omega$	0.16	$L_{si1}/uH$	9.53
$L_{p1}, L_{s1}/nH$	2.61	$R_{si1}^c/k\Omega$	0.29
$R_{p1}, R_{s1}/k\Omega$	299	$R_{si1}^c/k\Omega$	2.27e4
$C_{p1}, C_{s1}/fF$	1.77	$C_{si1}/fF$	1.47

Table 1: The determined model parameters

**Model Verification**

To verify the model's performance, an on-chip interleaved transformer with PGS (patterned ground shield) has been fabricated in a 0.5µm L50G CMOS process; the line width and spacing are 10 and 2µm respectively, and the transformer's inner diameter is 130µm, as shown in Figure 4. The measured S-parameter is extracted with a two-step (open and short) procedure to remove the undesired pad parasitics.

We used Z parameters converted from the measured S-parameters to determine the self-inductance ( $L_p$  and  $L_s$ ), quality factors ( $Q_p$  and  $Q_s$ ) and coupling coefficient (k) as functions of frequency. These are calculated using the following expressions:

$$L_p = \frac{im(Z_{11})}{2\pi f} \quad (11)$$

$$L_s = \frac{im(Z_{22})}{2\pi f} \quad (12)$$

$$Q_p = \frac{im(Z_{11})}{re(Z_{11})} \quad (13)$$


$$Q_s = \frac{im(Z_{22})}{re(Z_{22})} \quad (14)$$

$$k = \frac{\sqrt{im(Z_{12})im(Z_{21})}}{\sqrt{im(Z_{11})im(Z_{22})}} \quad (15)$$

To verify the procedure's efficiency and accuracy, we have compared the modelled and measured  $L_p$ ,  $L_s$ ,  $Q_p$ ,  $Q_s$  and k. It only takes four parallel branches (three branches for real poles and one for complex poles) to represent the high-order effects. The model parameters generated by the direct determination procedure are listed in Table 1.

For clarity in Figures 5 and 6, we compared the curves of  $-L_s$  and  $-Q_s$  instead of  $L_s$  and  $Q_s$ . RMS deviations of the simulated  $Q_p$  and  $Q_s$  in Figure 5 are below 2.59%, the simulated  $L_p$  and  $L_s$  in Figure 6 are below 6.89%, and the simulated k value in Figure 7 is 7.61%.

The frequency range chosen for the RMS calculation is between 0.1GHz and 20GHz. The relative deviation is below 1.13% for the DC inductance L, and below 3.47% for the peak value of Q. As demonstrated in these three figures and deviations, an excellent agreement has been found over a broad frequency range; hence, the model in Figure 1 is accurate enough to model the transformer, with the parameter-determining procedure showing high efficiency and accuracy without any optimisation.

The equivalent circuit model is compatible with common circuit simulators such as SPICE, for transient and frequency domain simulation. 

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